The Canadian Macroeconomy and the Yield Curve: An Equilibrium-Based Approach

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July 13, 2006

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Abstract

The authors develop and estimate an equilibrium-based model of the Canadian term structure of interest rates. The proposed model incorporates a vector-autoregression description of key macroeconomic dynamics and links them to those of the term structure, where identifying restrictions are based on the first-order conditions that describe the representative investor's optimal consumption and portfolio plan. A remarkable result is that the in-sample average pricing errors obtained with the equilibrium-based model are only slightly larger than those obtained with a far more flexible no-arbitrage model. The gains associated with parsimony become obvious out-of-sample, where the equilibrium model delivers much more accurate predictions, especially for yields with longer-term maturities. The preferred equilibrium model has impulse responses that are consistent with long-term inflation expectations being anchored, so a surprise increase in inflation does not necessarily raise expectations of higher future inflation.

JEL classification: E43, E44, E47, E52

Résumé

Les auteurs élaborent et estiment un modèle d'équilibre de la structure des taux d'intérêt canadiens, dans lequel la dynamique des principales variables macroéconomiques est représentée sous une forme vectorielle autorégressive et reliée à celle de la structure des taux. Les contraintes d'identification du modèle découlent des conditions du premier ordre qui définissent le plan optimal de consommation et de placement de l'investisseur représentatif. Résultat frappant, l'erreur moyenne de prévision des prix obtenue en échantillon est à peine plus élevée dans le modèle d'équilibre que dans un modèle beaucoup plus souple fondé sur l'absence d'arbitrage. Les gains découlant du caractère parcimonieux du modèle sont très nets au delà de la période d'estimation : le modèle d'équilibre produit des prévisions de qualité bien supérieure hors échantillon, surtout dans le cas des taux d'intérêt à long terme. Les profils de réaction que génère le modèle d'équilibre privilégié cadrent avec un ancrage des attentes d'inflation à long terme, en ce sens qu'une hausse imprévue de l'inflation n'accentue pas nécessairement les attentes d'une augmentation de l'inflation dans l'avenir.

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1. Introduction

Models of the term structure of interest rates have been mostly formulated in continuous time and in an arbitrage-free framework. Typically, bond yields are affine functions of a number of state variables that capture the uncertainty present in the economy. In many specifications, the state variables are unobserved. Econometrically, the latent factors are extracted from bond prices or yields by either assuming that a few bonds are priced perfectly by the model or by filtering techniques if all bonds are assumed to be priced with error. When three factors are specified, they are often interpreted as the level, slope, and curvature of the yield curve, following Litterman and Scheinkman (1991). Dai and Singleton (2003) and Piazzesi (2003) provide thorough surveys of this class of models.

Recently, several researchers have added observable macroeconomic variables to the latent factors to try to understand the channels through which the economy influences the term structure, and not simply describe or forecast the movements of the term structure. Ang and Piazzesi (2003) and Ang, Dong, and Piazzesi (2004) introduce measures of inflation and real activity as macroeconomic factors. The joint dynamics of these macro factors and the latent factors are captured by vector-autoregression (VAR) models, where identifying restrictions are based on the absence of arbitrage. Models with more macroeconomic structure have also been proposed recently by Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2004), and Bekaert, Cho, and Moreno (2003). These models combine the affine arbitrage-free dynamics for yields with a New Keynesian macroeconomic model, which typically consists of a monetary policy reaction function, an output equation, and an inflation equation.

In each of the aforementioned models, risk premiums for the various sources of uncertainty are obtained by specifying time-varying prices of risk that transform the risk-factor volatilities into premiums. The prices of risk, however, are estimated directly from the data without accounting for the fact that investors' preferences and technology may impose some constraints between these prices. Indeed, according to Diebold, Piazzesi, and Rudebusch (2005), "the goal of an estimated no-arbitrage macro-finance model specified in terms of underlying preference and technology parameters (such that the asset pricing kernel is consistent with the macrodynamics) remains a major challenge."

In this paper, we propose an equilibrium-based model that goes some distance towards this goal. We price bonds in an economy where investors derive utility from consumption and an external reference level of consumption. The new feature of this approach—introduced by Garcia, Renault, and Semenov (2002)—is that the reference level is formed by expectations about aggregate per capita consumption and not by looking at past consumption, as in habit models. Therefore, the growth rate of this reference level of consumption, which is what matters for pricing purposes, is made a function of contemporaneous and past variables that are deemed relevant. In our model, the short-term interest rate is considered a main explanatory variable, since we consider it a policy instrument under the influence of the central bank. We also relate the reference level of consumption to inflation and past consumption growth, in order to both capture persistence and measure real activity, and finally to the return on a stock index, in order to link the equity and the bond markets. This forecasting equation for consumption growth, to which a structural preference role is given, is added to other equations for the explanatory variables to form a VAR. The same preference parameters that affect the reference-level growth rate in the stochastic discount factor (SDF) impose restrictions on the pricing kernel, and therefore on the term premiums of bonds at various maturities. In Piazzesi (2003), affine general-equilibrium models are specified with preference shocks that are related to state variables, as in Campbell (1986) and Bekaert and Grenadier (2003). Wachter (2006) also proposes a consumption-based model of the term structure of interest rates, where nominal bonds depend on past consumption growth through habit, and on expected inflation. This model is essentially the same as the habit model of Campbell and Cochrane (1999), but the sensitivity function of the surplus consumption to innovations in consumption is chosen so as to make the risk-free rate a linear function of the deviations of the surplus consumption from its mean. Moreover, Wachter calibrates her model so as to make the nominal risk-free rate in the model equal to the yield on a three-month bond at the mean value of surplus consumption. This model has some similarities with ours, but our modelling for the reference level of consumption is more general and we estimate the model as in the no-arbitrage literature, allowing for a direct comparison.

The dynamic interaction between the macroeconomy and the term structure is explored by Diebold, Rudebusch, and Aruoba (2006) in a Nelson-Siegel empirical model of the term structure, complemented by a VAR model for real activity, inflation, and a monetary policy instrument. They find that the causality from the macroeconomy to yields is much stronger than in the reverse direction. Our model will allow for such an effect of macroeconomic variables on yields, but not the reverse. Allowing for the reverse effect complicates the estimation considerably. Ang, Dong, and Piazzesi (2004) use Markov chain Monte Carlo methods to allow for bidirectional linkages between the macroeconomy and the yields, while imposing no-arbitrage restrictions.

We start by estimating a first-order VAR comprising the short-term rate of interest, the return on the Toronto Stock Exchange (TSX) composite index, the rate of inflation, and the rate of consumption growth. We use a sample of quarterly data from the first quarter of 1962 to the first quarter of 2004. Given the parameter estimates of the VAR model and the Euler conditions for the prices of bonds at some chosen maturities, we can estimate the preference parameters by minimizing the least-square distance between the observed yields and the theoretical yields. In order to incorporate information from the various ends of the yield curve, we choose three maturities—2, 8, and 20 quarters—and use the available yields data from the first quarter of 1986 to the last quarter of 2002. Note that the available yields data begin much later (1986) than the macroeconomic data (1962).

For comparison purposes, we also estimate a no-arbitrage model similar to that used by Ang and Piazzesi (2003) and Ang, Piazzesi, and Wei (2006). In the latter, the authors use an approach that is similar to the method just described, but in a no-arbitrage framework.[2] They first estimate the VAR and use the estimated parameters together with the no-arbitrage bond-yield formulas to estimate the prices of risk that minimize a distance between the theoretical yields and the observed yields. By using the same VAR for the macroeconomic variables, we will be able to assess the relative contributions of the different modelling strategies. The equilibrium approach involves far fewer second-step parameters than the no-arbitrage approach. Therefore, one can expect that the no-arbitrage approach will perform better in-sample than the equilibrium approach, and that the reverse will be true out-of-sample, especially for longer maturities. When we assess the in-sample and out-of-sample pricing errors associated with the two methods, our results conform to this expected behaviour. What is more surprising is the overall good performance of the equilibrium model: the absolute pricing errors in-sample are very close to the errors of the no-arbitrage model, despite a large difference in the number of estimated parameters between the two models. It should be stressed that, in the equilibrium model, we maintain that bond prices are determined by the same preferences at short and long horizons, true to the spirit of a structural model. If the goal was simply to reduce pricing errors, we could adjust different preference parameters for each horizon.

To conclude the empirical assessment of the equilibrium model, we use the VAR specification for the dynamics of the macroeconomic variables to compute impulse-response functions for the yields and a long-short spread. We can then study the impact of an

[2]Ang, Piazzesi, and Wei (2006) propose such a sequential estimation strategy. What we gain in flexibility by proceeding in such a sequential manner, we may lose in efficiency of the estimators. Joint estimation is possible, but will add a significant layer of complexity.

innovation in, say, the inflation rate on the yield structure. A striking feature is the highly persistent effects that shocks have on the yield curve. In each case, the effects are seen to persist for more than 20 quarters before showing signs of significant mean reversion. As one might expect, shocks to the short rate and the inflation rate produce the highest responses. The impulse responses are consistent with a monetary reaction that raises the short end of the yield curve in response to positive shocks to output and inflation. Our results reveal that the short end of the yield curve is more sensitive than the long end to such reactions.

The rest of this paper is organized as follows. Section 2 describes the equilibrium model with a reference level of consumption that will be used to price bonds. We also specify the dynamics of the macroeconomic variables that will influence the yields. Section 3 is dedicated to model estimation and evaluation. We specify the benchmark no-arbitrage model, the data sources, and the econometric method used to estimate the parameters and ultimately to compute the yields. We report the pricing errors for the various specifications as well as the impulse-response functions for the equilibrium model. Section 4 offers some conclusions.

2. An Equilibrium Model with a Reference Level

Most models of the term structure are specified in a no-arbitrage setting, where the link between the objective data-generating measure and the risk-neutral measure is specified exogenously and is not tied to preference parameters. Piazzesi (2003) describes affine general-equilibrium models within the context of a representative-agent endowment economy. Models in this category[3] are represented by a utility function where the agent consumes an endowment process and receives exogenous preference shocks. These shocks are tied to a vector of state variables. As we will show below, this representation is in the spirit of our model with a reference level of consumption.

We adopt a consumption-based asset pricing proposed by Garcia, Renault, and Semenov (2002), whereby the investor derives utility from consumption relative to some reference consumption level as well as from this level itself. The one-period real SDF

[3]Piazzesi (2003) refers in particular to Campbell (1986), Bekaert and Grenadier (2003), and Wachter (2006). In each of these papers, a stochastic process is specified for consumption or surplus consumption as defined in Campbell and Cochrane (1999).

defined by this model is

$$m_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}}{S_t}\right)^{\gamma-\varphi}, \tag{1}$$

where δ is the time preference parameter and γ is the curvature parameter for consumption C_t relative to S_t, a time-varying reference consumption level. The parameter φ controls the curvature of utility over this benchmark level. The future evolution of the reference level is constrained to coincide with real aggregate per capita consumption, $\bar{C}$, in terms of conditional expectations:

$$E_t[S_{t+h}] = E_t[\bar{C}_{t+h}], \text{ for all } h \geq 0. \tag{2}$$

Investors can include in their assessment macroeconomic variables that belong to their information set at time $t+h$. In nominal terms, the one-period SDF is given by

$$m^{\$}_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{S_{t+1}}{S_t}\right)^{\gamma-\varphi} \left(\frac{P^c_{t+1}}{P^c_t}\right)^{-1}, \tag{3}$$

where P^c_{t+1}/P^c_t is the gross rate of inflation between periods t and $t+1$.

An important part of the modelling strategy is to determine the variables that investors consider for characterizing the growth rate of the reference level of consumption:

$$s_{t+1} = a + x_{t+1}b, \tag{4}$$

where $s_{t+1} = \log S_{t+1}/S_t$ and x_{t+1} represents a vector of variables, current and past, deemed to forecast the growth of the reference level. To obtain an estimate of this growth rate, estimates of a and b are needed along with the values of the variables x_{t+1}. Using the equality of expectations in (2) at horizon 1, we can write a regression equation in terms of consumption growth:

$$c_{t+1} = a + x_{t+1}b + \varepsilon_{t+1}, \tag{5}$$

where $c_{t+1} = \log C_{t+1}/C_t$. Note that the bar over real aggregate consumption is left out, since it is the same as the consumption of the representative investor. Garcia, Renault, and Semenov (2002) show that, by using different specifications for the reference level, they can recover the SDFs associated with the three main strands of consumption-based asset-pricing models: habit formation (Constantinides 1990; Campbell and Cochrane 1999), recursive utility (Epstein and Zin 1989), and loss aversion (Barberis, Huang, and Santos 2001). Models of consumption with a reference level may be rationalized by a behavioural model, as in Kőszegi and Rabin (2006).

For the purpose of pricing bonds, we assume that the investor's forecasts of consumption growth are based on

$$c_{t+1} = b_0 + b_1 R_t^s + b_2 R_t^m + b_3 \pi_t + b_4 c_t + \varepsilon_{t+1}, \tag{6}$$

where $R_t^m = \log P_t^m / P_{t-1}^m$ represents the (log) return on the market portfolio, $\pi_t = \log P_t^c / P_{t-1}^c$ is the rate of inflation, and R_t^s is the short-term interest rate. For the purpose of modelling the term structure of interest rates, the inclusion of the short rate is obviously essential. It captures the fact that the short rate is a policy instrument under the influence of the central bank. Instead of using directly the Bank Rate, which has the behaviour of a step function, we prefer to include the yield to maturity of a one-period bond; that is, $R_t^s = -\log P_t^s$, where P_t^s is the bond price. Inclusion of inflation might be important to price nominal bonds and longer-term maturities. The other variables (c_t, R_t^m) are also potentially important as they provide a way to include a more traditional, strictly consumption-based reference level (as in external habit models) and a link to equity markets. Although we do not price equities in the current paper, such an equation for the growth of the reference level could be used to price equities (see Garcia, Renault, and Semenov 2002).

The joint dynamics governing the evolution of the explanatory variables that appear in (6) are modelled as a first-order vector autoregression (VAR) in $Y_t = (R_t^s, R_t^m, \pi_t, c_t)'$. Ang, Piazzesi, and Wei (2006) also specify a VAR in terms of observables, and include the short rate, the term spread, and output growth. The uniqueness of the Cholesky decomposition used to compute impulse-response functions is with respect to the model-consistent ordering of the variables within the vector Y_t. The first-order VAR that governs the evolution of these variables is written as

$$Y_t = \mu + \Phi Y_{t-1} + \varepsilon_t, \tag{7}$$

where the error terms are i.i.d. $N(0, \Omega)$, with $\Omega = E[\varepsilon_t \varepsilon_t'] = \Sigma\Sigma'$. Given the specification of the reference level via (6), the SDF can be rewritten as

$$m_{t+1}^{\$} = \delta \exp(b_0 \kappa) \left(P_t^s\right)^{-b_1 \kappa} \left(\frac{P_t^m}{P_{t-1}^m}\right)^{b_2 \kappa} \left(\frac{P_t^c}{P_{t-1}^c}\right)^{b_3 \kappa} \left(\frac{P_{t+1}^c}{P_t^c}\right)^{-1} \left(\frac{C_t}{C_{t-1}}\right)^{b_4 \kappa} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}, \tag{8}$$

where $\kappa = \gamma - \varphi$. Note that the SDF in (8) is well defined, in that it can only be positive. By chaining together the one-period SDFs, we find that the price of a nominal bond that pays one dollar at time $t + n$ is given by

$$B(t,t+n) =$$

$$E_t\left[\delta^* \left(P^s_{t,t+n-1}\right)^{-b_1\kappa} \left(\frac{P^m_{t+n-1}}{P^m_{t-1}}\right)^{b_2\kappa} \left(\frac{P^c_{t+n-1}}{P^c_{t-1}}\right)^{b_3\kappa} \left(\frac{P^c_{t+n}}{P^c_t}\right)^{-1} \left(\frac{C_{t+n-1}}{C_{t-1}}\right)^{b_4\kappa} \left(\frac{C_{t+n}}{C_t}\right)^{-\gamma}\right], \tag{9}$$

where

$$\delta^* = \delta^n \exp(nb_0\kappa), \tag{10}$$

$$\left(P^s_{t,t+n-1}\right)^{-b_1\kappa} = \exp(b_1\kappa I_1 \sum_{i=0}^{n-1} Y_{t+i}) \tag{11}$$

$$\left(\frac{P^m_{t+n-1}}{P^m_{t-1}}\right)^{b_2\kappa} = \exp(b_2\kappa I_2 \sum_{i=0}^{n-1} Y_{t+i}) \tag{12}$$

$$\left(\frac{P^c_{t+n-1}}{P^c_{t-1}}\right)^{b_3\kappa} = \exp(b_3\kappa I_3 \sum_{i=0}^{n-1} Y_{t+i}), \tag{13}$$

$$\left(\frac{P^c_{t+n}}{P^c_t}\right)^{-1} = \exp(-I_3 \sum_{i=1}^{n} Y_{t+i}), \tag{14}$$

$$\left(\frac{C_{t+n-1}}{C_{t-1}}\right)^{b_4\kappa} = \exp(b_4\kappa I_4 \sum_{i=0}^{n-1} Y_{t+i}), \tag{15}$$

$$\left(\frac{C_{t+n}}{C_t}\right)^{-\gamma} = \exp(-\gamma I_4 \sum_{i=1}^{n} Y_{t+i}), \tag{16}$$

with $I_1 = [1,0,0,0]$, $I_2 = [0,1,0,0]$, $I_3 = [0,0,1,0]$, and $I_4 = [0,0,0,1]$. Multiplying (10) through (16) together and taking the expectation conditional on time-t information, we find that bond prices are given by

$$B(t,t+n) = \delta^* \exp\left(M_\mu \mu + M_Y Y_t + \frac{1}{2}\sigma^2\right), \tag{17}$$

where

$$\begin{aligned}
M_\mu &= M_1 + J_1, \\
M_Y &= (J_1 + J_2)\sum_{i=1}^{n-1} \Phi^i + J_1\Phi^n + J_2, \\
M_1 &= \sum_{i=1}^{n-1}\left[(J_1+J_2)\sum_{j=i}^{n-1}\Phi^{n-j-1} + J_1\Phi^{n-i}\right], \\
\sigma^2 &= M_1\Omega M_1' + J_1\Omega J_1',
\end{aligned}$$

with $J_1 = [0, 0, -1, -\gamma]$ and $J_2 = \kappa[b_1, b_2, b_3, b_4]$; Φ^0 is set equal to the 4×4 identity matrix. The model-implied n-period yield is thus given by

$$\hat{y}_t^n = \begin{cases} -\log B(t, t+n)/n & \text{if} \quad 0 \leq B(t, t+n) \leq 1, \\ 0 & \text{if} \quad B(t, t+n) > 1. \end{cases} \tag{18}$$

The developed partial-equilibrium model does not explicitly incorporate money, so a priori it does not exclude negative interest rates. This can be explained by the fact that if the cost of storing currency exceeds that of storing other financial assets, then nominal interest rates could become negative. On the other hand, if the cost of storing cash is zero and non-monetary assets are viewed as perfect substitutes, then nominal interest rates cannot become negative. We impose the lower zero bound on bond yields in (18) since, in reality, negative nominal interest rates occur very rarely.[4]

The modelling of the reference level may appear arbitrary, since one chooses the variables that may determine the benchmark level of consumption. Whichever variables are chosen, however, they enter the SDF in a restricted way, since they are affected by a common preference expression ($\kappa = \gamma - \varphi$), which imposes testable restrictions on asset prices. One can use the usual Euler conditions on several asset returns together with equation (5) to infer the preference parameters. One can also test the model with a J-test or an asymptotically more appropriate test that accounts for weak instruments (see Stock and Wright 2000 and Yogo 2004). Garcia, Renault, and Semenov (2002) estimate and test several such consumption-based capital-asset-pricing models with a reference level with returns on Treasury bills and on equities.

Two arbitrage-based models are close in spirit to our research agenda. Bansal and Zhou (2002) propose a model of the term structure of interest rates with regime switches.[5] Bekaert and Grenadier (2003) propose a bond- and stock-pricing model in an affine economy. We will sketch here the main difference between these two models and our equilibrium approach. Ignoring the regime-switching feature, Bansal and Zhou (2002) start with the Lucas (1978) model, a particular case of our model with $\gamma = \varphi$, where $M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$. The next step is to relate the SDF to the return of an asset that delivers the consumption stream: $R_{c,t+1} = (M_{t+1})^{-1}$. Bansal and Zhou (2002) then assume a stochastic process for

[4]One recent occurrence is the slightly negative interest rates on short-term Japanese government bonds in late 1998.

[5]In Garcia and Luger (2005), we generalized our equilibrium specification to accommodate potentially different dynamics for the macroeconomic variables across the business cycle, together with a forecasting model for recessions. Unfortunately, the value-added of that cyclical model was not apparent in the short sample over which the zero-coupon yields are available for Canada.

the logarithm of this return: $r_{c,t+1} = x_t + \frac{\lambda}{\sigma}\frac{x_t}{2} + \sqrt{x_t}\frac{\lambda}{\sigma}u_{t+1}$, where x_t is a latent variable following a Cox, Ingersoll, and Ross (1985) process. By absence of arbitrage and the normality of u_{t+1}, the return on a one-period safe asset enters the SDF as

$$M_{t+1} = \exp(-r_{ft} - (\frac{\lambda}{\sigma})^2\frac{x_t}{2} - \sqrt{x_t}\frac{\lambda}{\sigma}u_{t+1}). \tag{19}$$

The SDF for the economy with regime shifts is similar, except that the λ and σ are made state dependent. In a multivariate setting, Bekaert and Grenadier (2003) exploit a similar arbitrage-based model, but without regime switches, to price both bonds and stocks:

$$\begin{aligned} Y_{t+1} &= \mu + AY_t + \Sigma_{Y_t}\varepsilon_{t+1}, \\ m_{t+1} &= \mu_m + \Gamma_m^{'} Y_t + \Sigma_{m_t}\varepsilon_{t+1}, \end{aligned} \tag{20}$$

where Y_{t+1} is a vector of observable and latent state variables, the matrices Σ_{Y_t} and Σ_{m_t} are such that the processes for Y_t and m_t are a combination of Vasicek and square-root processes, and ε_{t+1} is standard normal. In their applications of this general specification, Bekaert and Grenadier (2003) include a dividend-growth process, a latent variable, and inflation in their state variables, and choose several specifications for m_t, including Campbell and Cochrane's (1999) habit-formation model.[6] In both these models, the latent process is used to capture the features of the short-rate process. Indeed, for estimation, Bekaert and Grenadier (2003) use the nominal interest rate to filter the latent process. In our model, the short rate enters directly into the SDF through the preferences. In Wachter (2006), who also uses the habit-based model of Campbell and Cochrane (1999), the equilibrium short-term interest rate is restricted to be a linear function of surplus consumption. It is further calibrated to be equal to the nominal interest rate in steady state, when surplus consumption equals its long-term mean.

Our model has a number of attractive features for capturing the yield curve. First, by making the short rate exogenous, the model captures the fact that central banks of industrialized countries may affect the short end of the yield curve. The model provides a way to introduce monetary policy in the form of reaction functions, which is consistent with equilibrium. Second, our model links bond prices with the real and monetary sides of the economy. Recently, several papers (Bekaert, Cho, and Moreno 2003; Hördahl, Tristani, and Vestin 2006; Rudebusch and Wu 2004) have appended a term-structure model to a New Keynesian macro model. Our proposed equilibrium approach to capture the dynamic interactions between the macroeconomy and the term structure is more parsimonious

[6]Although it is not our purpose here, Campbell and Cochrane's (1999) model could be accommodated in our general specification, as shown by Garcia, Renault, and Semenov (2002).

and is built only on observable variables. Moreover, the risk premiums are fully pinned down by the risk processes and the preference parameters, while they are left free in the arbitrage-based models of the term structure used in these papers. A third advantage of our model is that it provides a link between the stock market and the bond market by including a return on a stock market index in the equation that determines the growth of benchmark consumption. Fourth, as in any equilibrium model, we can price any asset and, in particular, derivatives such as swaps, futures, and options on interest rates.[7]

3. Model Estimation and Evaluation

3.1 Benchmark model

The described equilibrium model links the dynamics of the term structure of interest rates to macroeconomic variables. Ang and Piazzesi (2003) also establish such a link through a no-arbitrage model of the term structure. The equilibrium approach taken here and the no-arbitrage approach both incorporate macroeconomic dynamics by specifying a VAR. The fundamental difference lies in the way identification is achieved. In the approach advocated here, identifying restrictions are based on the first-order conditions that describe the representative investor's optimal consumption and portfolio plan. In Ang and Piazzesi's approach, identifying restrictions are based only on the absence of arbitrage.

Ang and Piazzesi (2003) and Ang, Piazzesi, and Wei (2006) assume that the SDF follows a conditionally log-normal process:

$$m_{t+1}^{\$} = \exp\left(-R_t^s - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right),$$

where λ_t are the time-varying market prices of risk associated with the sources of uncertainty, ε_t. The vector λ_t is a linear function of Y_t:

$$\lambda_t = \lambda_0 + \lambda_1 Y_t, \tag{21}$$

where Y_t is described by (7) so that λ_0 is a 4×1 vector and λ_1 is a 4×4 matrix.

Bond prices are given by

$$B^{na}(t, t+n) = \exp(A_n + B_n' Y_t), \tag{22}$$

[7]Garcia, Luger, and Renault (2003) estimate an equilibrium model with regime-switching and Epstein and Zin (1989) preferences using prices on Standard and Poor's 500 options.

where the coefficients A_n and B_n are defined recursively by

$$\begin{aligned} A_n &= A_{n-1} + B'_{n-1}(\mu - \Sigma\lambda_0) + \frac{1}{2}B'_n\Sigma\Sigma' B_n, \\ B_n &= (\Phi - \Sigma\lambda_1)' B_{n-1} - I'_1, \end{aligned}$$

with $I_1 = [1, 0, 0, 0]$. The initial conditions are $A_1 = 0$ and $B_1 = -I'_1$. The above definitions of A_n and B_n incorporate the no-arbitrage restrictions; see Ang and Piazzesi (2003) and Ang, Piazzesi, and Wei (2006) for details.

3.2 Data description

The macroeconomic data used to estimate the VAR are quarterly, covering the period 1962Q1 to 2004Q1. Consumption growth is based on seasonally adjusted total personal expenditures in chained 1997 dollars, and inflation is defined by the corresponding implicit chain prices. The returns on the market portfolio are proxied using the month-to-month changes in the (log) value of the TSX composite index; the log returns are then averaged to obtain quarterly returns. The short rate is also obtained by averaging the monthly 3-month Treasury bill rate.

The bond data consist of a set of daily zero-coupon yields on Canadian government bonds, obtained from the Bank of Canada. This data set was derived by Bolder and Gusba (2002) using the Merrill Lynch exponential spline model.[8] In the described yield-curve model, one period corresponds to one quarter. The bond data, therefore, were aggregated up to the quarterly frequency by averaging the daily yields. The result is a set of 68 quarterly observations, with available maturities of 2, 4, 8, 12, 16, and 20 quarters, from 1986Q1 to 2002Q4. Table 1 provides summary statistics of the yield data at the quarterly frequency.

3.3 Estimation results

Following Ang, Piazzesi, and Wei (2006), we estimate the model in two steps. In the first step, we estimate the VAR parameters μ, Φ, and Σ by ordinary least squares. The preference parameters that enter the bond-pricing equation are then estimated in the second step, conditional on the VAR estimates, by solving the non-linear least-squares

[8]Bolder, Johnson, and Metzler (2004) describe a similar database, which will be kept current and be publicly available on the Bank of Canada's website.

problem:

$$\min_{\{\delta,\gamma,\varphi,b\}} \sum_{t=1}^{T} \sum_{n=1}^{N} \left[\hat{y}_t^n - y_t^n\right]^2 , \tag{23}$$

where y_t^n is the market yield of an n-period bond at time t and $\hat{y}_t^n$ is the corresponding model-implied yield; the choice variables are the three preference parameters—δ, γ, and φ—and the parameters of the consumption reference level, $b = [b_0, b_1, b_2, b_3, b_4]$. The model-implied yields are computed according to (18) conditional on the estimated parameters of the macroeconomic fundamentals VAR model.

The no-arbitrage model is also estimated by solving a non-linear least-squares problem similar to the one in (23); i.e., conditional on the VAR estimates obtained in the first step, the sum of squared differences between actual yields and those implied by the no-arbitrage model, $-\log B^{na}(t, t+n)/n$, is minimized with respect to the parameters in λ_0 and λ_1.

Of course, this step-by-step estimation methodology does not deliver the most statistically efficient estimates. On the other hand, its computational simplicity is a considerable advantage, especially when the models need to be updated on a regular basis. In order to incorporate information from the various ends of the yield curve, we estimate the models based on the observed behaviour of the 2-, 8-, and 20-quarter yields. Figure 1 plots the time series of these three yields.

Table 2 reports the estimation results for the no-arbitrage model. Note that all the components of λ_0 appear to be statistically significant, while only the component of λ_1 associated with the short-term interest rate appears to be significant. Table 3 reports the estimation results for the equilibrium model. When $\varphi = 1$ in the equilibrium model, we obtain a preference specification where the ratio of the investor's consumption to the reference level is all that matters, as in habit formation models. If $\gamma \neq \varphi$ and $\varphi \neq 1$, then the investor takes into account both the ratio of his consumption to the reference level and this level itself when choosing how much to consume. The model is estimated for both the standard expected utility case ($\gamma = \varphi$) and the case where utility depends on a reference level of consumption. In these two cases, the preference parameters γ (and φ in the reference level case) are significantly different from 1. In the reference level model, all the coefficients in the consumption reference level, except for the one associated with lag consumption, are significant. The strong significance of the coefficient of the market return (b_2) in the reference level equation is worth noticing.

Table 4 reports summary statistics of the absolute pricing errors (in percentage points) for the various specifications considered. It is immediately clear that relaxing the standard expected utility constraint vastly improves the fit of the equilibrium model. This is not surprising, given that the short rate plays no role in the equilibrium bond-pricing formula under standard expected utility. The reference-level equilibrium model fares well against the no-arbitrage model, especially at longer maturities. This result is even more impressive when one considers that, in addition to the first-step VAR parameters, the equilibrium model has only 8 second-step parameters compared with the 20 second-step parameters needed for the no-arbitrage model with time-varying prices of risk. Figures 2–4 show the resulting in-sample fits for maturities of 2, 8, and 20 quarters, respectively.

The equilibrium and no-arbitrage models are further compared in terms of their one-quarter-ahead prediction abilities. For each quarter t, we estimate the VAR model and the two term-structure models using data up to and including quarter t, and then forecast the next quarter's yields using the VAR's forecasts for period $t+1$. Hence, we use only data available in the information set in period t when forming the forecasts for period $t+1$. Given that we need at least 20 observations to estimate the no-arbitrage model, prediction abilities are compared over the period 1991Q1–2002Q4, resulting in 48 one-quarter-ahead forecasts.

The out-of-sample forecasts are also compared to those from a simple random walk. Ang and Piazzesi (2003) find that the forecasting performance of a VAR with no-arbitrage restrictions is slightly better than a random walk. Table 5 reports summary statistics of the one-quarter-ahead absolute forecast errors (in percentage points). For maturities of 2 and 4 quarters, the no-arbitrage model outperforms the equilibrium model and performs like the random walk. Although neither model beats the random walk at all maturities, the more parsimonious equilibrium model outperforms the no-arbitrage model for all maturities greater than 8 quarters. Further, the forecasting performance of the equilibrium model tends to mimic that of the random walk for longer maturities. On the contrary, the performance of the no-arbitrage model diverges from the random walk towards the long end of the yield curve.

Some insight into the poor forecasting behaviour of the no-arbitrage model can be gained by noting that even when there is no systematic risk because the variables in the state vector Y_t of the macroeconomic fundamentals VAR model are i.i.d. ($\Phi = 0$), the implied risk premium will still be time-varying whenever $\lambda_1 \neq 0$ in (21). Further, that variation over time is proportional to the variances of the variables in Y_t. Duffee (2002) shows that traditional affine term structure models produce forecasts that are typically worse than forecasts produced by simply assuming that future yields are equal to current yields.[9] Duffee explains that the poor forecasting performance of those traditional models

[9] See Egorov, Hong, and Li (2006) for related evidence.

is due to the fact that the implied compensation for risk is a multiple of the variance of the state vector. This tight link makes it difficult to replicate some stylized facts of historical excess bond returns. Duffee concludes that, for the purpose of forecasting, traditional affine term structure models are largely useless. On the contrary, the dynamics of the risk premium captured by the equilibrium model seem to do better in this regard.

3.4 Impulse responses of yields

The Fisher relation indicates that economic shocks could affect nominal interest rates by altering real interest rates, expectations of inflation, or both. Since shocks may not always move real rates and inflation components in the same direction, the response of nominal rates cannot be determined a priori. We examine the steady-state dynamics implied by the equilibrium model by means of impulse-response functions. A standard Cholesky decomposition can be used to identify "structural shocks," which in turn are used to compute impulse-response functions for the VAR in the usual way. Yield-curve impulse-response functions are then computed by feeding the VAR responses into the equilibrium bond-pricing formula.

Figures 5–7 show the impulse responses of yields to one percentage point shocks to the short rate, R^s, the return on the market portfolio, R^m, the rate of inflation, π, and the rate of consumption growth, c. The steady-state 2-quarter yield is 7.823 per cent, while that of the 20-quarter yield is 7.49 per cent. These values imply a slightly inverted steady-state yield curve, as shown by the responses in Figure 7 (the horizontal line represents the steady-state slope). A striking feature of Figures 5–7 is the highly persistent effects the various shocks have. In each case, the effects are seen to persist for more than 20 quarters before showing signs of significant mean reversion. A shock to the short rate has the greatest effect, followed by shocks to inflation, then consumption, and finally the return on the market portfolio.

Monetary policy actions are often believed to be transmitted to the economy through their effect on market interest rates. According to this view, a monetary policy tightening pushes up both short- and long-term interest rates, leading to less spending by interest-sensitive sectors of the economy (such as housing, consumer durable goods, and business fixed investments) and therefore to lower economic growth. Conversely, a monetary policy easing results in lower interest rates that stimulate economic activity. Although casual observation suggests a close connection between monetary actions and short-term interest rates, the relationship between policy and long-term interest rates appears more elusive. One reason for this is that the effect of policy actions on long-term interest rates (via the

Fisher relation) will depend in part on how they impact inflation expectations. The policy interest rate can be linked to long-term interest rates through the expectations hypothesis, which states that a long-term rate is equal to the average of a short-term rate expected to prevail over the maturity of the long-term asset plus a risk premium. According to this view, policy actions affect long-term rates by changing current and expected future short-term rates.

The impulse responses are consistent with a monetary reaction that raises the short end of the yield curve in response to positive shocks to consumption—a large share of GDP—and inflation. This result echoes the now standard policy rule of Taylor (1993); i.e., the monetary policy maker exerts an upward (downward) influence on interest rates in response to indications of cyclical expansions (slowdowns). Figures 5 and 6 show that the short end of the yield curve is more sensitive than the long end to such reactions. Indeed, the more negatively sloped yield curve suggested by Figure 7 is due to the greater increase in shorter-term yields. Such responses are consistent with long-term inflation expectations being anchored, so a surprise increase in inflation, for example, does not necessarily raise expectations of higher future inflation. Over the 1990s, the Bank of Canada put considerable effort into making the conduct of monetary policy more transparent to financial market participants. Several of the initiatives were designed to help market participants better understand the Bank's monetary reaction function. Muller and Zelmer (1999) discuss the benefits of reducing market uncertainty about the central bank's reaction function and show that the Bank of Canada's efforts at increasing transparency appear to have helped market participants anticipate pending monetary policy actions. The results presented here also suggest that the Bank of Canada enjoys a fairly large degree of credibility and transparency, which concurs with the Bank's explicit inflation-control targeting policy. It is interesting to note that in the case of the United States—which does not have an explicit inflation target—Diebold, Rudebusch, and Aruoba (2006) find that surprise increases in inflation boost future inflation expectations.

4. Conclusion

Most term-structure models have been formulated in a no-arbitrage setting where bond yields are affine functions of a state vector. In the first generation of models, the short rate was the only state variable in the economy. Recently, term-structure models have modelled the dynamics of bond yields jointly with the dynamics of some key macroeconomic variables. For example, Ang, Piazzesi, and Wei (2006) use GDP growth along with the short rate and a term spread variable to estimate the dynamics of the economy with a quarterly VAR. Bond yield risk premiums are captured by market prices of risk that are linear functions of the state vector. This is tantamount to an exogeneity assumption that ignores the underlying preferences of investors.

We have proposed an equilibrium model that is very close in spirit to this setting, but where risk premiums are determined by the preferences of the representative investor. This specification is based on a model where the investor derives utility with reference to a benchmark consumption level, as in habit-formation models. However, our specification for the dynamic evolution of this reference level is not determined solely by past consumption, as in habit-based equilibrium models. In our model, the growth rate of benchmark consumption is a function of the short rate, inflation, a stock market return, and past consumption growth.

Preferences impose tight restrictions on the SDF and deliver in the end a more parsimonious model than the arbitrage-free model. In our setting, we need to estimate eight second-step parameters compared with twenty for the arbitrage-free model. A remarkable result is that the average pricing errors obtained in-sample with the equilibrium model are only slightly larger than the errors obtained with the more flexible arbitrage-free model. The gain associated with parsimony appears out-of-sample, where the equilibrium model provides much smaller errors for yields with maturities longer than one year in a one-quarter-ahead rolling forecast exercise. It should be emphasized that the model is built to make estimation and forecasting very easy and robust. This feature makes the model particularly attractive for current analysis and policy simulations.

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Table 1. Summary Statistics of Yield Data

	Maturity in quarters					
	2	4	8	12	16	20
Mean	0.066	0.067	0.069	0.071	0.072	0.074
Std. deviation	0.027	0.025	0.022	0.021	0.019	0.019
Skewness	0.47	0.37	0.28	0.23	0.18	0.15
Kurtosis	2.23	2.12	1.95	1.82	1.74	1.69
Min	0.021	0.025	0.032	0.037	0.041	0.043
Max	0.12	0.13	0.12	0.12	0.11	0.11

Note: There are 68 quarterly observations of the yields from 1986Q1 to 2002Q4.

Table 2. Parameter Estimates: No-Arbitrage Model

Parameter	Estimate			
λ_0	-0.52	(0.0030)	[-173.33]	
	-0.20	(0.073)	[-2.74]	
	0.86	(0.045)	[19.11]	
	-0.093	(0.043)	[2.16]	
λ_1	2.14	0.14	-0.42	0.33
	(0.25)	(1.17)	(2.16)	(3.41)
	[8.56]	[0.12]	[-0.19]	[0.10]
	-0.32	-0.28	-0.61	-0.45
	(1.15)	(5.99)	(10.01)	(15.89)
	[-0.27]	[-0.05]	[-0.06]	[-0.03]
	-0.37	-0.11	2.05	-0.29
	(0.69)	(3.74)	(5.99)	(9.52)
	[-0.53]	[-0.03]	[0.34]	[-0.03]
	-0.38	0.65	-1.13	-0.59
	(0.69)	(3.61)	(6.00)	(9.53)
	[-0.55]	[0.18]	[-0.18]	[-0.06]

Notes: Standard errors appear in parentheses and t-statistics appear in brackets.

Table 3. Parameter Estimates: Equilibrium Model

Parameter	Estimate	Standard error	t-statistic
Standard expected utility case			
δ	0.95	0.013	73.26
$\gamma(=\varphi)$	4.76	0.52	7.29
Reference level model			
δ	0.99	0.0032	316.87
γ	2.73	0.11	15.48
φ	2.13	0.12	9.77
κ	0.60	0.011	53.45
b_0	0.031	0.0076	4.06
b_1	-1.74	0.079	-21.45
b_2	0.15	0.042	3.70
b_3	0.91	0.43	2.09
b_4	0.53	0.54	0.98

Notes: The t-statistics for γ and φ are for the null hypothesis that the parameter equals one. The other t-statistics in the table are for the null hypothesis that the corresponding parameter equals zero.

Table 4. Absolute Pricing Errors (Percentage Points)

	Maturity in quarters					
	2	4	8	12	16	20
No-arbitrage model						
Mean	0.21	0.38	0.52	0.56	0.60	0.69
Std. dev.	0.22	0.36	0.47	0.48	0.50	0.51
Min	0	0.02	0	0	0	0.08
Max	1.05	1.56	2.02	1.96	2.09	2.22
Equilibrium model						
Standard expected utility case						
Mean	3.35	3.01	2.72	2.59	2.55	2.53
Std. dev.	2.13	2.07	2.08	2.06	2.05	2.06
Min	0.04	0.05	0.05	0.02	0.02	0
Max	8.63	9.14	8.61	8.06	7.70	7.45
Reference level model						
Mean	0.54	0.48	0.51	0.56	0.65	0.77
Std. dev.	0.39	0.35	0.41	0.48	0.53	0.56
Min	0.01	0	0.02	0	0	0.02
Max	1.70	1.33	1.72	2.05	2.23	2.38

Note: Values less than 10^{-2} are reported as zero.

Table 5. One-Quarter-Ahead Absolute Forecast Errors (Percentage Points)

	Maturity in quarters					
	2	4	8	12	16	20
No-arbitrage model						
Mean	0.51	0.55	0.72	0.85	0.95	1.02
Std. dev.	0.42	0.46	0.48	0.49	0.53	0.60
Min	0.02	0.01	0	0.02	0.03	0.04
Max	1.83	2.04	2.07	2.02	2.46	2.88
Equilibrium model						
Mean	1.13	0.88	0.65	0.57	0.55	0.54
Std. dev.	0.49	0.42	0.47	0.48	0.48	0.50
Min	0.08	0.14	0	0	0.02	0.03
Max	2.15	1.92	2.02	2.12	2.11	2.08
Random walk						
Mean	0.57	0.54	0.49	0.46	0.42	0.39
Std. dev.	0.51	0.50	0.48	0.45	0.42	0.40
Min	0.01	0.01	0.01	0.02	0.01	0
Max	2.28	2.34	2.37	2.26	2.08	1.90

Note: Values less than 10^{-2} are reported as zero.

Figure 1. The solid, dashed, and dotted lines represent the 2-, 8-, and 20-quarter yields, respectively.

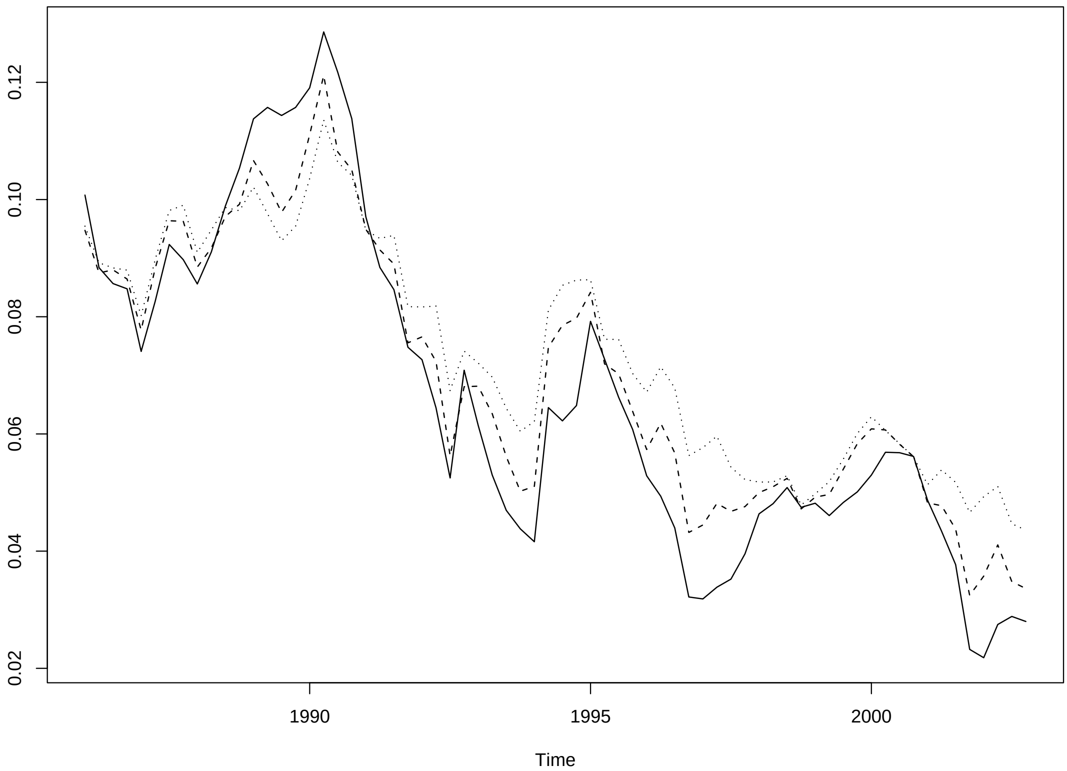

Figure 2. The solid and dashed lines represent the actual and fitted 2-quarter yields. The top panel represents the no-arbitrage model, and the bottom panel represents the equilibrium model.

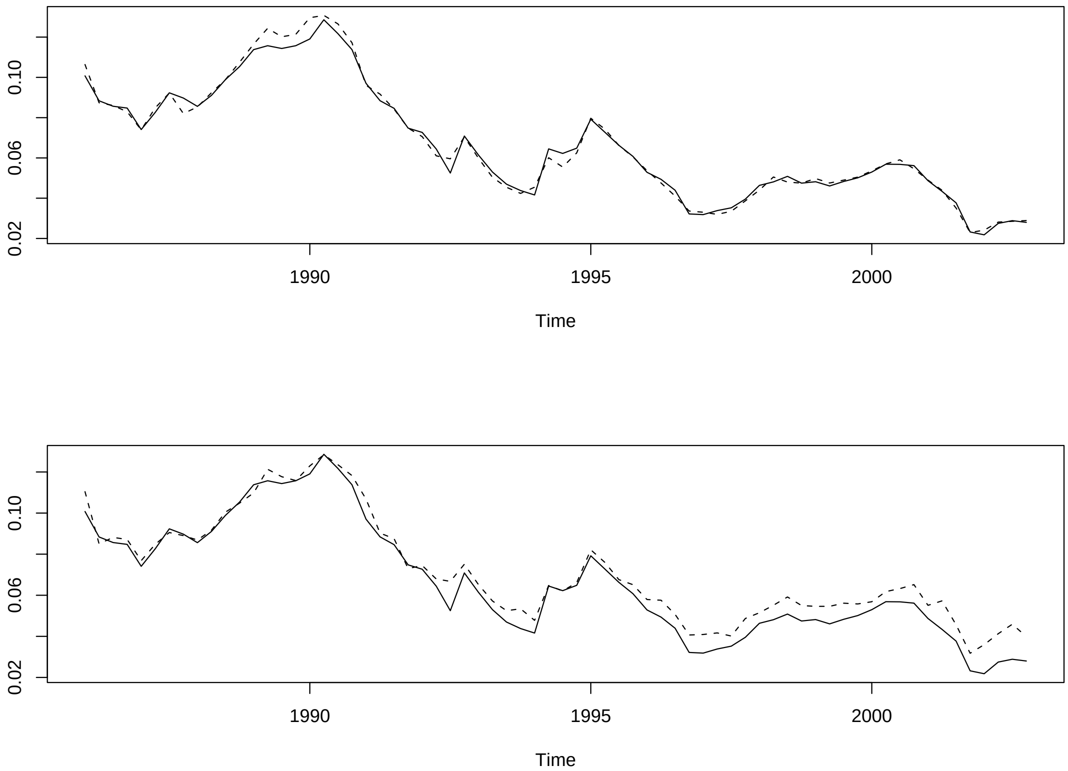

Figure 3. The solid and dashed lines represent the actual and fitted 8-quarter yields. The top panel represents the no-arbitrage model, and the bottom panel represents the equilibrium model.

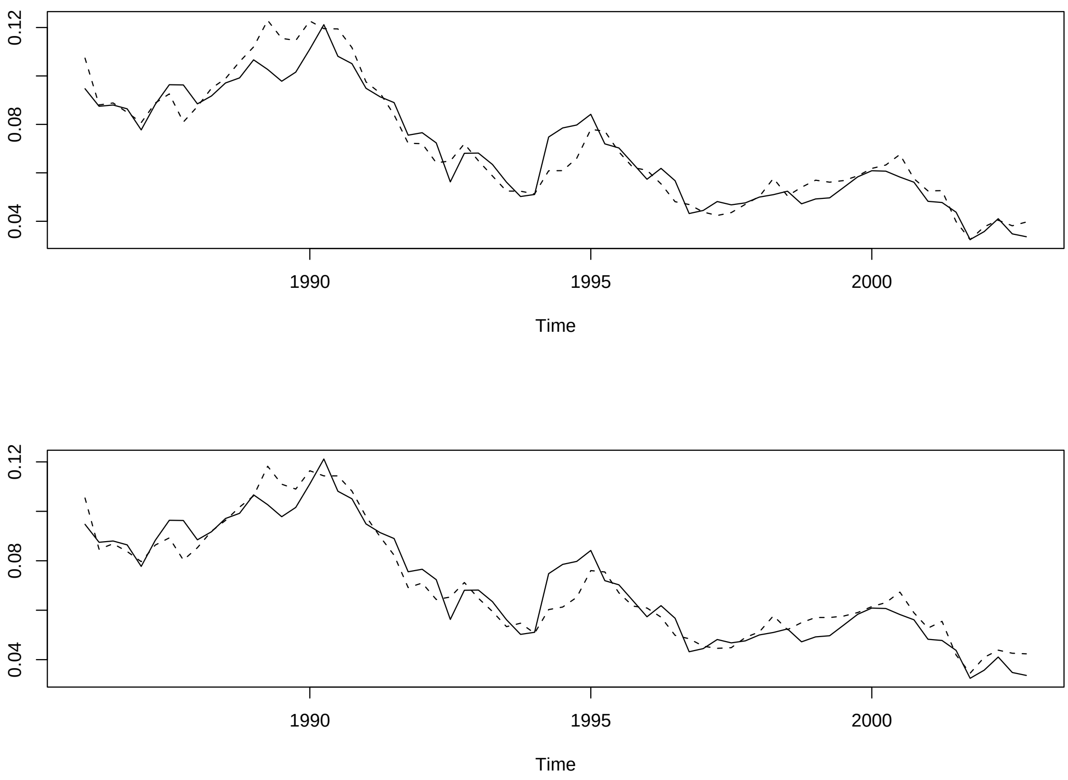

Figure 4. The solid and dashed lines represent the actual and fitted 20-quarter yields. The top panel represents the no-arbitrage model, and the bottom panel represents the equilibrium model.

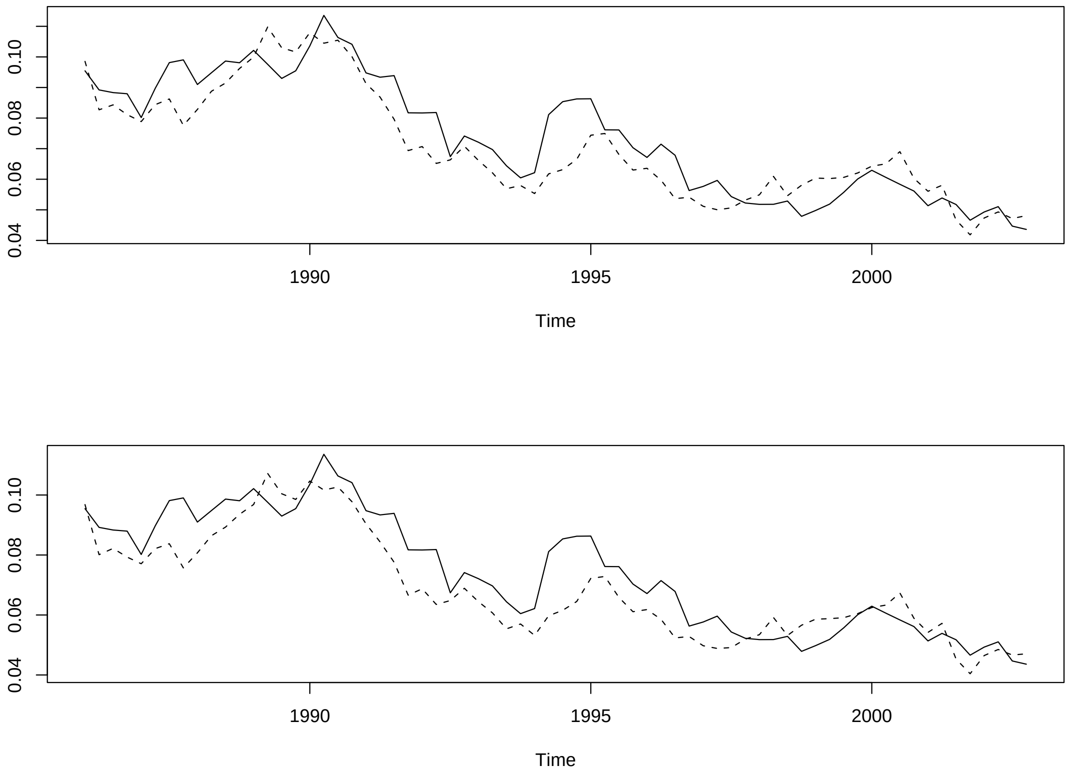

Figure 5. Impulse responses of 2-quarter yield.

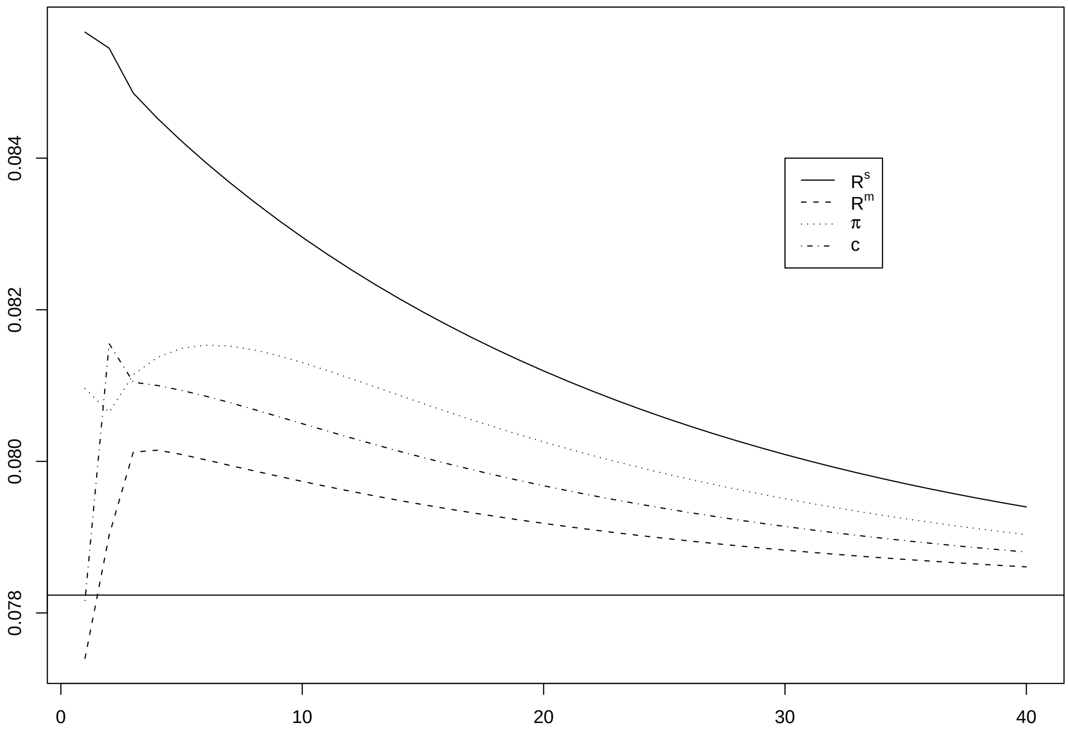

Figure 6. Impulse responses of 20-quarter yield.

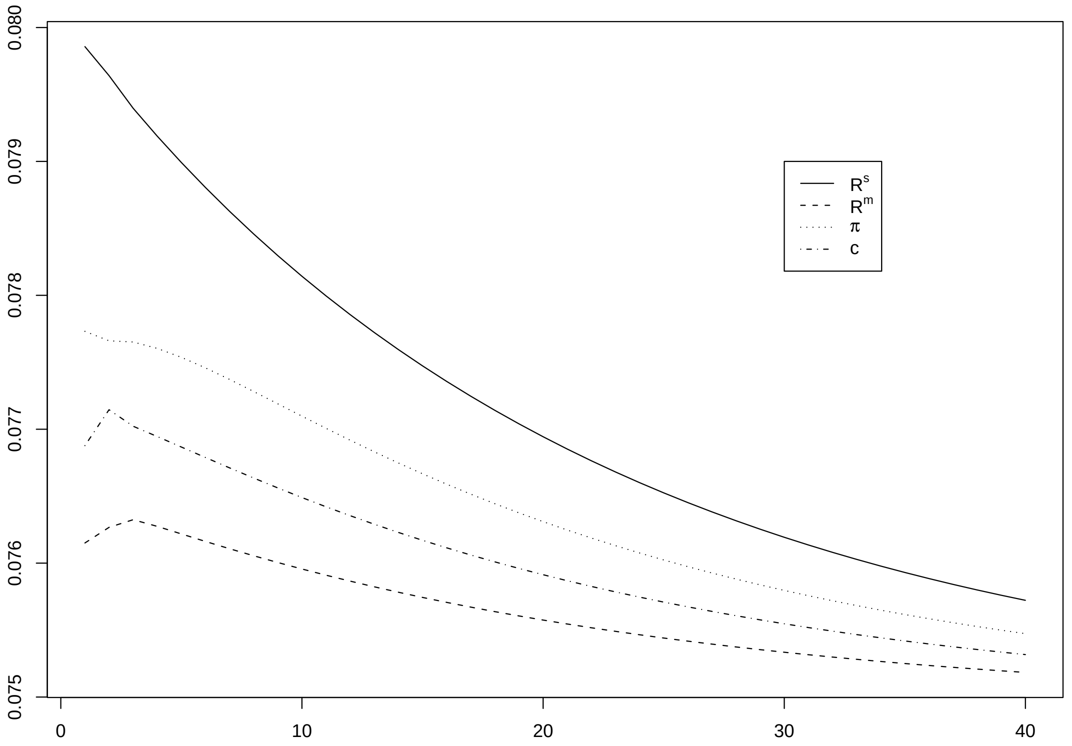

Figure 7. Impulse responses of spread between 20- and 2-quarter yields.