Macro-Prudential Policy Coordination and Global Regulatory Spillovers

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Job Market Paper

November 2011

Abstract

This paper analyzes the extent to which achieving the financial stability objective of banking regulation requires international policy coordination. We study a two-country model of systemic liquidity risk-taking, where pecuniary externalities and financial market imperfections provide a rationale for macro-prudential regulation. Curbing risk-taking via regulation lowers the price of liquidity during crises and thereby reduces the costs associated with market incompleteness. But it also distorts productive investment decisions. Optimal regulation therefore trades off an improvement in exchange efficiency with a deterioration in production efficiency. But while the production efficiency costs are incurred domestically, the exchange efficiency benefits are global in nature due to pecuniary externalities operating across borders. Absent international coordination, this results in national authorities regulating their financial systems too lightly in an attempt to free-ride on foreign liquidity provision when a crisis materializes. The theory also outlines a channel by which tighter regulation in a given country induces more risk-taking abroad and implies that national macro-prudential policies are strategic substitutes across countries.

Keywords: Systemic crises, international coordination, macro-prudential regulation, interbank markets

JEL Codes: F36, G21, F42, E44, D62

*I am greatly indebted to Anton Korinek and Enrique Mendoza for their invaluable guidance. I am also particularly grateful to John Shea and Carlos Vegh for their support. For useful suggestions and comments, I thank Tobias Adrian, Boragan Aruoba, Giancarlo Corsetti, Marco Del Negro, Pablo D'Erasmo, Thomas Eisenbach, Linda Goldberg, Christian Grisse, John Haltiwanger, Rebecca Hellerstein, Todd Keister, Ricardo Lagos, Steve Laufer, Ben Mandel, Paolo Pesenti, Asani Sarkar, David Skeie, Jean Tirole and Tanju Yorulmazer. I am also grateful to the Federal Reserve Bank of New York for its hospitality and support. All errors are my own. E-mail: bengui@econ.umd.edu

1 Introduction

The financial globalization process that started in the early 1980s was accompanied early on by attempts to harmonize banking regulation internationally. The original motivation for the first Basel accords (Basel I) was twofold: to ensure the stability of the global financial system and to eliminate distortions to competition arising from heterogenous regulatory regimes. Over time, national competitiveness concerns turned more dominant (Tarullo 2008), and they became the focus of the academic literature analyzing the international linkages relevant for banking regulation (see e.g. Acharya 2003, Dell Arricia and Marquez 2006). This focus on competitiveness issues raises the question of whether the goal of financial stability *in and of itself* requires an international coordination of banking regulation. This paper investigates this question in a model where agents' contribution to systemic risk calls for a *macro-prudential* approach to regulation.

We refer to liquidity as the aggregate amount of resources set aside to satisfy potential needs for funds. Liquidity has public goods properties during periods of market stress (Shin 2010). By limiting agents' exposure to liquidity risk, regulators can reduce the scarcity of liquidity during crises and alleviate credit crunches. But national regulators fail to adequately internalize the share of positive externalities associated with global liquidity that operates across borders. Consequently, individual countries try to free-ride on the foreign provision of liquidity. A lack of international coordination therefore results in insufficient macro-prudential regulation. This underprovision of ex-ante regulation leads to more severe and more costly financial crises ex-post.

The analysis is undertaken in the context of a two-country version of a model of liquidity demand, in the spirit of Holmstrom and Tirole (1998) and Caballero and Krishnamurthy (2001).[1] Ex-ante identical agents invest in risky long-term projects that may require an additional liquidity injection along the road. Liquidity shocks are imperfectly correlated across countries, implying opportunities for international risk-sharing. Because of moral hazard, cross-country insurance against these shocks is limited, but agents can set aside liquid resources ex-ante by investing in a short-term asset (i.e. self-insure), or alternatively, they can borrow ex-post on an international spot market up to some limit. In this environment, market incompleteness results in a constrained inefficiency of the competitive equilibrium. Agents fail to internalize that their collective investment choice affects the severity of a potential credit crunch, and they underinvest in short-term assets in equilibrium. Curbing agents' exposure to liquidity risk via prudential regulation can restore constrained efficiency. We compare three alternative allocation mechanisms: (a) the laissez-faire outcome (competitive equilibrium), (b) the constrained efficient outcome achieved by setting regulation cooperatively at a global level, and (c) the equilibrium of a policy game where regulation is chosen non-cooperatively by welfare maximizing national authorities.

The inefficiency of decentralized investment decisions in the model results from a pecuniary externality operating via the price of global liquidity, i.e. the international interest rate, in a crisis. This interest rate depends on the scarcity of liquidity. A marginal increase in the liquidity of the

[1] See also the related consumer liquidity demand models of Diamond and Dybvig (1983), Jacklin (1987), Bhattacharya and Gale (1987), Hellwig (1994), von Thadden (1999) and others.

representative agent's ex-ante investment portfolio from its competitive equilibrium level lowers the interest rate and causes a redistribution of wealth from lenders to borrowers in a crisis. Due to the incompleteness of markets, borrowers value liquidity more highly than lenders ex-post. Since ex-ante, any agent could end up with a high or with a low valuation of liquidity, a marginally lower interest rate achieves a redistribution of resources from low valuation states of nature to high valuation states. It partially substitutes for missing risk markets and leads to a first order welfare gain. A global planner maximizing a representative agent's welfare chooses to require agents to tilt their investment in favor of short-term assets, with the consequence of alleviating credit crunches when a crisis occurs. But while regulation brings about an improvement in exchange efficiency, it distorts productive investment decisions away from their competitive level. In choosing the optimal extent of regulation, a global planner thus trades off an improvement in exchange efficiency with a deterioration in production efficiency in the world economy.

National planners, who set regulation non-cooperatively to maximize the welfare of a domestic representative agent, do perceive the dependence of the severity of a potential credit crunch upon ex-ante investment choices. But in contrast to a global planner, they attempt to shift surplus in favor of residents rather than restore constrained efficiency. In particular, national planners internalize that more domestic ex-ante liquidity hoarding (i.e. more domestic investment in short-term assets resulting from tighter regulation) would alleviate a foreign credit crunch by lowering the interest rate and leading to a redistribution of resources in favor of foreign residents in states of nature where foreigners with a high valuation of liquidity borrow from domestic lenders with a low valuation of liquidity. But they derive no benefits from alleviating credit crunches abroad. Since the pecuniary externality operates across borders, national planners do not internalize the full exchange efficiency benefits of regulation. At the same time, the production efficiency costs of regulation are incurred domestically. Consequently, national planners generally fall short of imposing the optimal extent of regulation. This underprovision of ex-ante regulation results in more severe and more costly financial crises ex-post, in the form of larger interest rate spikes and more forced liquidation of real investments. In fact, national planners' incentives to manipulate state-contingent terms of trade (i.e. interest rates) in favor of residents can even result in the equilibrium of the regulation game featuring less liquid and more risky investments than the laissez-faire benchmark. When this occurs, welfare is lower with uncoordinated regulation than under laissez-faire. In other words, uncoordinated regulation can be worse than no regulation at all.

The underprovision of regulation can be interpreted as a beggar-thy-neighbor outcome. Global liquidity mainly benefits distressed countries during crises, so it is attractive for a country to take advantage of foreign liquidity provision when one is distressed, but not to provide liquidity to distressed foreigners when one is intact. Viewed through the lens of the literature on international policy coordination (see e.g. Cooper 1985 and Persson and Tabellini 1995), the result can also be interpreted as arising from the attempt by national planners to make use of monopoly and monopsony power in the market for international liquidity during crises. Alternatively, the underprovision of regulation can be seen as resulting from a variation of the hold up problem (Grout 1984 and Tirole 1986). In the model, the returns from "investing" in a sound domestic financial system (via prudential regulation) within the trading relationship between the two countries exceed those outside it. And once the investment is sunk (the regulation enacted), the investor (regulator) has to share the gross return with its trading partner (the other country). As in the classical hold up problem, this expropriation eventuality leads to an underinvestment ex-ante.

The constrained inefficiency of competitive equilibria in incomplete markets economies is well known since the work of Hart (1975), Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). So too is the suboptimality of uncoordinated macroeconomic policies since at least Johnson (1965) and Hamada (1976). The novelty of the present paper is to analyze in a common framework the interplay between distortions arising from market incompleteness and those resulting from openness and countries' monopoly and monopsony power in global markets. The analysis outlines the close link between the mechanics of policy incentives arising from these two kinds of distortions. A constrained global planner internalizes pecuniary externalities in much the same way as strategically acting governments - or more generally, agents - do. But the former does so to improve exchange efficiency and reduce the cross-sectional wedges between marginal rates of substitutions caused by incomplete markets. In contrast, the latter use market power to shift surplus in their favor, generally at the cost of widening these wedges and reducing overall efficiency.

The model offers predictions about the international spillover effects of changes in regulation. Starting from the laissez-faire, the introduction of a small regulation in one country increases risk-taking in the other country. The transmission channels work through the lowering of interest rates during crises brought about by the extra amount of liquidity set aside in the regulated country. The model also predicts that macro-prudential policies are strategic substitutes across countries, as a country's tightening of regulation, by increasing the amount of global liquidity, reduces the benefits of regulation for the other country. Finally the model delivers the surprising result that, starting from the laissez-faire, a unilateral introduction of regulation can be welfare reducing for the regulated country and welfare improving for the unregulated country.

The analysis is done in the context of a model where agents can borrow internationally when hit by liquidity shocks. We show that all results fully carry over to a setup where agents raise liquidity during crises by selling long-term assets rather than by borrowing. In that setup, the pecuniary externality works through an asset price and takes the form of a fire-sale externality in the presence of cash-in-the-market pricing (Allen and Gale 1998). The coordination problem between regulators can hence be alternatively interpreted as arising from a failure to mutually commit to supporting asset prices during crises.

At an abstract level, the regulation game analyzed in this paper corresponds to a liquidity demand model in which two large agents have market power. It is often argued that market power mitigates the harm caused by systemic externalities, because large agents partly internalize the effect of their actions on prices. We show that this need not be the case. Whether market power attenuates or amplifies the distortions caused by market incompleteness crucially depends on the direction of the relevant pecuniary externalities, as well as on whether they are imposed on ex-post identical or ex-post different agents. In situations, similar to our model, where agents exert positive externalities on ex-post different agents, market power might quite naturally amplify distortions.

Literature

The paper fits into a recent research agenda that motivates financial regulation from a second best perspective in incomplete markets environments. It is most closely related to the liquidity regulation approach of Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009), and to the sudden stop prevention analysis of Caballero and Krishnamurthy (2001, 2004) for emerging countries. As in these papers, the market failure calling for government intervention in the present paper originates from a pecuniary externality operating on a spot market for liquidity.[2] But our paper stands out from these by explicitly formulating a multi-country framework. The model structure is therefore closer to that of Castiglionesi, Feriozzi, and Lorenzoni (2010), whose main focus is on the positive implications of financial integration for the liquidity of banks' portfolios and the size of interest rate spikes in crises. They briefly touch upon normative issues by solving numerically for the constrained efficient allocation, but ignore the potential coordination problem that arises in attempting to implement this allocation when regulation is set at a national level. In contrast, we consider the incentives faced by rationally acting national regulators, analytically characterize the equilibrium of the policy game and compare it with both the laissez-faire outcome and the constrained efficient allocation achieved by coordinating regulation globally.

In modeling the strategic interaction among national governments, this paper follows the game theoretic approach to macroeconomic policy coordination pioneered by Hamada (1974, 1976, 1979).[3] Governments are assumed to set policies to maximize national welfare, while taking into account their power to affect international prices. By attempting to use market power to shift surplus in favor of domestic residents, governments generally end up in Pareto-inferior equilibria. Our results parallel those of studies where distortions induced by openness can overturn the direction of a desirable policy intervention, such as Corsetti and Pesenti (2001). In their model, monopoly distortions in production together with nominal rigidities make unexpected monetary expansions welfare improving (as in Blanchard and Kiyotaki 1987 and Ball and Romer 1989) when carried out simultaneously at home and abroad. But the same policy can be welfare reducing for a country acting in isolation because of adverse endogenous terms of trade movements. Similarly, in our model, financial market imperfections make prudential regulation unambiguously welfare improving when introduced jointly at home and abroad. But terms of trade movements working through the interest rate during crises and associated spillover effects can make the unilateral introduction of such a policy welfare reducing for a given country.

Our paper is also related to Acharya (2003), who studies the consequences of an international convergence of bank capital requirements in the presence of heterogenous national closure policies,

[2] Lorenzoni (2008) and Korinek (2011b) emphasize a related pecuniary externality working through asset prices and fire-sale spirals during financial crises. Bianchi (2011), Jeanne and Korinek (2011), Bianchi and Mendoza (2010) and Bengui (2011) investigate the quantitative relevance of these pecuniary externalities in infinite horizon macroeconomic models.

[3] A immense literature on macroeconomic coordination and interdependence ensued in the late 1970s, 1980s and 1990s, with e.g. Frenkel and Razin (1985), van Wijnbergen (1986) and Frenkel, Razin, and Sadka (1991) for fiscal policy, and e.g. Rogoff (1985), Canzoneri and Henderson (1994, 1991) for monetary policy. See the reviews in Cooper (1985) and Persson and Tabellini (1995).

and Dell Arricia and Marquez (2006), who analyze the incentives for national bank regulators to form a regulatory union. But in both papers, regulatory spillovers operate through changes in the degree of competition faced by banks on international loan markets during tranquil times. In our model, this competition channel is absent and international spillovers only arise through pecuniary externalities operating in a global market for liquidity during financial crises. It therefore stands out in focussing on an international coordination motive directly linked to the financial stability objective of banking regulation.

The paper is structured as follows. The model is presented in section 2. Global and national regulations are analyzed in section 3. Section 4 works out the implications of the framework for the international spillovers of regulatory policies. Section 5 presents an alternative model in which the ex-post intermediation of funds occurs via an asset market rather than via a credit market. Section 6 presents a brief numerical illustration, and section 7 concludes.

2 A two-country model of liquidity demand

This section presents the model in which the coordination problem between national regulators is analyzed. The environment is laid down in section 2.1, and the competitive equilibrium is characterized in section 2.2.

2.1 Preferences, technology and markets

Agents, Time and Preferences The world economy is composed of two countries, indexed by $j \in \{A, B\}$, that are ex-ante identical with respect to preferences, endowments and technology. Each country is populated by a continuum of identical agents. Time lasts for three periods $t = 0, 1, 2$, and consumption takes place at date 2. Agents' preferences over date 2 consumption are represented by an increasing, concave and twice continuously differentiable utility function $u(\cdot)$.

Technology Each agent is born with an endowment of one unit of the consumption good at date 0, and decides how to allocate this endowment between investment in a risky and illiquid project k and investment in a safe storage technology ℓ. With probability $1 - \alpha$, all projects in country j remain *intact*. An intact project does not require additional funds at date 1 and yields a date 2 return of $A > 1$. With probability α, a project becomes *distressed* and necessitates a renewed investment of one good per unit at date 1 for the project to yield a date 2 return of A. Each unit not shored up at date 1 yields a reduced date 2 return of $r < 1$. Distressed agents have the possibility to scale down investment at date 1. For an initial project of size k, a continuation scale θ results in a date 1 cost of θk and a date 2 return of $rk + \theta \Delta k$, where $\Delta \equiv A - r$. The storage technology yields one date $t+1$ good per date t unit invested, and can be accessed both at date 0 and date 1.

Uncertainty All the uncertainty is resolved at date 1. Liquidity shocks are imperfectly correlated across the two countries. The sample space is given by $\Omega = \{(i, i), (i, s), (s, i), (s, s)\}$, where in state

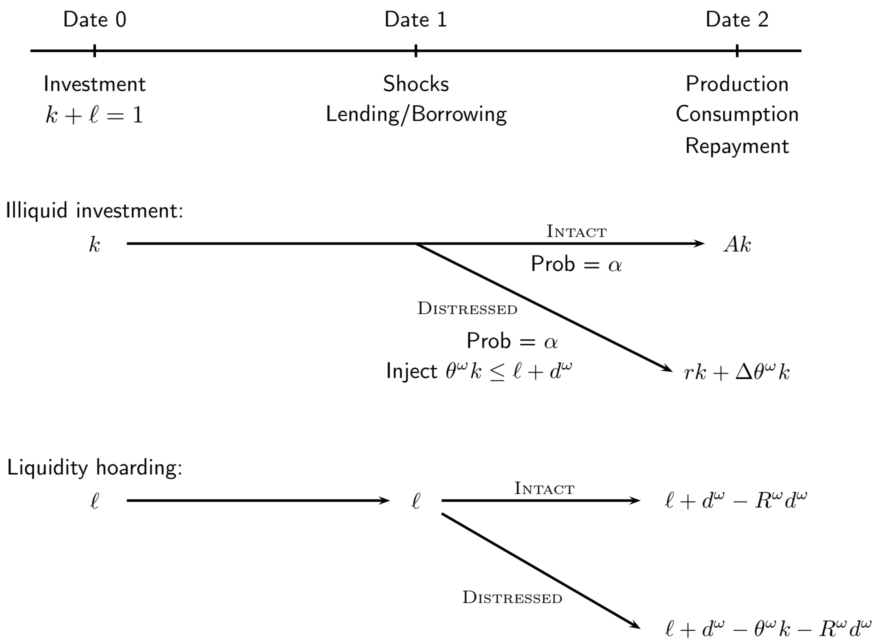

Figure 1: Time line.

(i, s) country A is intact and country B is distressed.[4] The probability mass function $\pi : \Omega \rightarrow [0, 1]$ assigns a probability to each state $\omega \in \Omega$.

Markets The imperfect correlation of liquidity shocks across countries creates opportunities for international risk-sharing. The assumption maintained throughout the paper, though, is that only non-state contingent bonds can be traded across borders. This can be motivated by the lack of verifiability of country-specific shocks.[5]

Agents can substitute for risk-sharing by borrowing and lending at date 1 on an international spot market at interest rate R^ω. Distressed agents borrow d_j^ω, while intact agents lend $-d_j^\omega$. The size of loans on this market is constrained by distressed agents' ability to commit to repay their debt at date 2. More precisely, the date 1 investment continuation decision is assumed to be private information and only a fraction κ of distressed agents assets can be seized by international lenders. This implies that distressed agents' borrowing is limited by a collateral constraint given by $R^\omega d_j^\omega \leq \kappa r k_j$. The time line is represented in Figure 1.

[4]In assuming representative national residents, we follow the common practice in open-economy macroeconomics of focusing on country-specific sources of risk (see Obstfeld and Rogoff 1996). This can be justified by the evidence for more risk-sharing prevailing within than between countries (Atkeson and Bayoumi 1993, Bayoumi and Klein 1995, Crucini 1999).

[5]Because agents are ex-ante identical, non-contingent claims as well as claims contingent on the states (i, i) and (s, s) would not be traded at date 0 in equilibrium, so these markets can be abstracted from for the sake of simplicity.

Assumptions on parameters

Assumption 1 (Yield of illiquid project)**.** *The yield on the illiquid project satisfies* $1 < A \leq \frac{3}{2}$ *and* $\Delta \equiv A - r > 1$.

The assumption that $A > 1$ ensures that the illiquid project has a higher yield than the liquid asset in normal times, albeit not excessively so. The assumption that $\Delta > 1$, on the other hand, illustrates the idea that distressed firms have a high marginal value of investment during crisis times. It implies that shoring up an additional unit of a distressed project, if feasible, is always socially desirable.

Assumption 2 (Probability of crisis)**.** *The probability* α *of a project becoming distressed satisfies* $\underline{\alpha} < \alpha < \bar{\alpha}$, *where* $\underline{\alpha}$ *is given by*

$$\underline{\alpha} = \frac{(A-1)u'(A)}{(A-1)u'(A) + (\Delta - r)u'(r)}$$

and $\bar{\alpha}$ *is the smallest positive root of the quadratic equation*

$$[(1-\alpha)^2+\rho\alpha(1-\alpha)](A-1)u'\left(\frac{2A}{3}+\frac{1}{3}\right)+(1-\rho)\alpha(1-\alpha)(A-\Delta)u'\left(\frac{2A}{3}+\frac{\Delta}{3}\right)+\alpha(r-\Delta)u'\left(\frac{2r}{3}+\frac{\Delta}{3}\right) = 0.$$

Assumption 2 captures the fact that financial crises are low probability events, but that they are likely enough to induce precautionary behavior in the form of some liquidity hoarding. $\alpha > \underline{\alpha}$ guarantees that crises are likely enough that agents find it optimal to hoard a positive amount of liquid assets, while $\alpha < \bar{\alpha}$ ensures that crisis are rare enough that when a crisis hits one of the two countries, the global aggregate amount of liquidity hoarded ex-ante is not sufficient to shore up all illiquid projects in the distressed country. Together, these two assumptions imply that there is partial liquidation in a crisis.

2.2 Competitive equilibrium

We start by considering equilibrium on the date 1 spot market for given date 0 decisions. We will then proceed backwards to solve for the competitive equilibrium and regulated equilibria at date 0.

2.2.1 Date 1 spot market equilibrium

The date 1 value of an intact agent in country j is given by

$$V_i^\omega(k_j, \ell_j) \equiv \max_{0 \leq -d_j^\omega \leq \ell_j} u\Big(Ak_j - d_j^\omega(R^\omega - 1) + \ell_j\Big). \tag{1}$$

The intact agent's date 2 consumption in (1) is given by the sum of the return on its illiquid project Ak_j, the return on the loan made on the date 1 spot market $-R^\omega d_j^\omega$, and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^\omega$. Without loss of generality, we assume that intact agents can only lend on the date 1 spot market. Their lending capacity is limited by their date 1 liquid resources ℓ_j.

The form of the objective in (1) implies a simple loan supply schedule for intact agents. For $R^\omega < 1$, intact agents do not want to lend at all. At $R^\omega = 1$, they are indifferent between lending any amount between 0 and ℓ_j. Finally, when $R^\omega > 1$ they are willing to lend all their available liquidity ℓ_j.

The date 1 value of a distressed agent in country j is given by

$$V_s^\omega(k_j, \ell_j) \equiv \max_{\theta_j^\omega, d_j^\omega} u\Big(rk_j + \Delta\theta_j^\omega k_j - \theta_j^\omega k_j + \ell_j - (R^\omega - 1)d_j^\omega\Big) \tag{2}$$

subject to

$$\theta_j^\omega k_j \leq \ell_j + d_j^\omega \tag{3}$$

$$R^\omega d_j^\omega \leq \kappa r k_j \tag{4}$$

$$\theta_j^\omega \leq 1 \tag{5}$$

A distressed agent's date 2 consumption in (2) is the sum of the return on its illiquid project $rk_j + \Delta\theta_j^\omega k_j$ and the return on the funds invested at date 1 in the storage technology $\ell_j + d_j^\omega - \theta_j^\omega k_j$, minus the debt repayment $R^\omega d_j^\omega$. (3) is the date 1 budget constraint indicating that reinvestment $\theta_j^\omega k_j$ needs to be covered by the sum of ex-ante liquidity hoarding ℓ_j and ex-post borrowing d_j^ω. (4) is a collateral constraint, and (5) indicates that investment cannot be scaled up at date 1.

Given the assumption that $\Delta > 1$ and the fact that θ_j^ω and d_j^ω enter additively in the expression of consumption, the loan demand and optimal continuation scale of distressed agents take simple forms. For $R^\omega < 1$, the agents hit their collateral constraint (4). For $1 \leq R^\omega < \Delta$, they borrow the minimum of the amount they need to salvage all their assets, $k_j - \ell_j$, and their borrowing limit, $\kappa r k_j / R^\omega$. At $R^\omega = \Delta$, they are indifferent between borrowing any amount between 0 and $\min\Big\{k_j - \ell_j, \kappa r k_j/\Delta\Big\}$. Finally, for $R^\omega > \Delta$, they do not want to borrow at all. The optimal continuation θ_j^ω is accordingly given by $\min\Big\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{R^\omega}\Big\}$ for $R^\omega < 1$, by any amount between $\frac{\ell_j}{k_j}$ and $\min\Big\{1, \frac{\ell_j}{k_j} + \frac{\kappa r}{\Delta}\Big\}$ when $R^\omega = 1$, and by $\min\Big\{1, \frac{\ell_j}{k_j}\Big\}$ for $R^\omega > \Delta$. The loan supply schedule of intact agents and the loan demand schedule of distressed agents are displayed in Figure 2. In the right panel, the loan demand curve is drawn for different values of κ, with the curves to the left associated with a lower κ.

Assumptions 1 and 2 guarantee that agents do not find it optimal to hoard more liquidity at date 0 than what is needed to shore up their own entire project were they to become distressed at date 1. The date 1 spot market equilibrium is therefore simply given by $R^\omega = 1$ and $d_A^\omega = d_B^\omega = 0$ in state (i, i), and by $R^\omega = \Delta$ and $d_A^\omega = d_B^\omega = 0$ in state (s, s).

In states of world where one country is intact and the other is distressed, i.e. in (i, s) and (s, i), the spot market equilibrium can a priori fall in four distinct regions, depending on where the loan demand and loan supply curves intersect, similarly to Caballero and Krishnamurthy (2001).

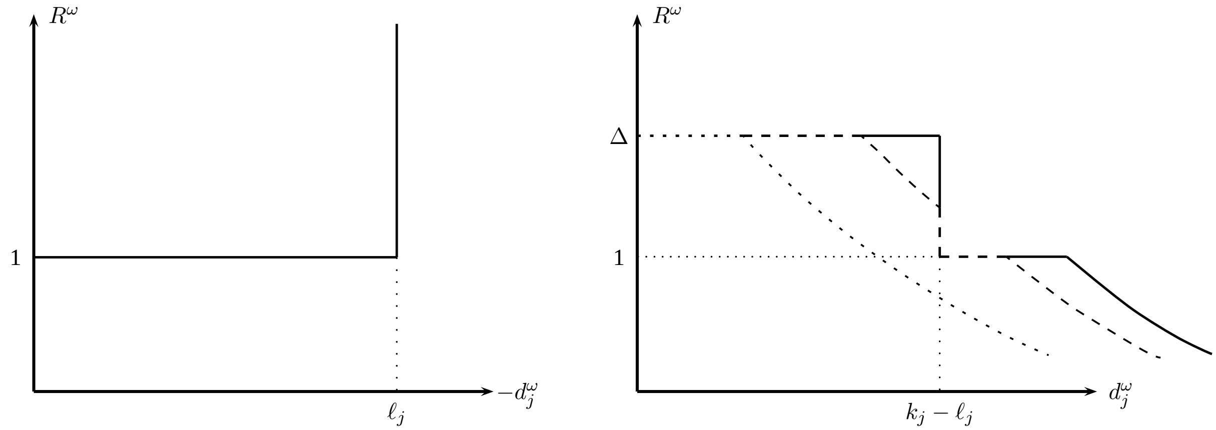

Figure 2: Supply by intact (left) and demand by distressed (right) on date 1 spot market.

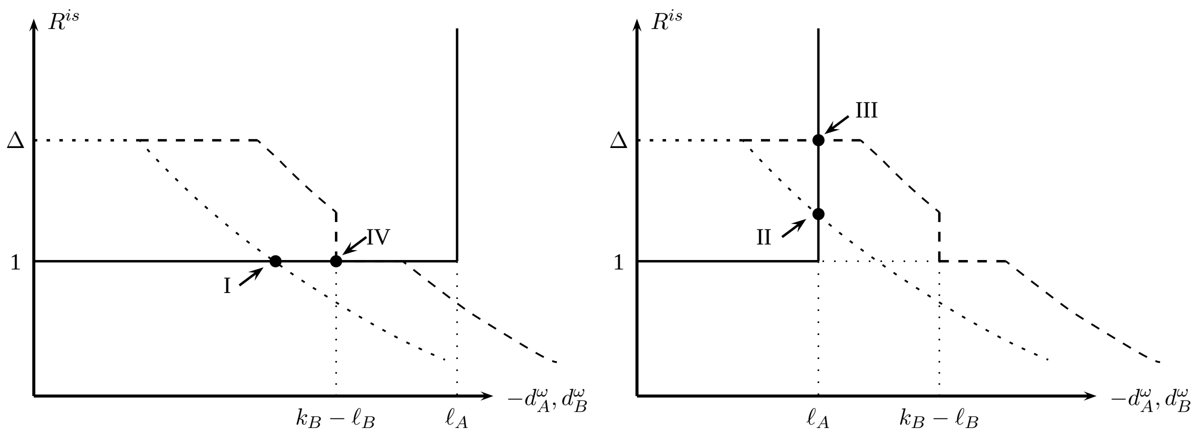

Figure 3: Regions for spot market equilibrium in state (i, s).

		Global liquidity	
		ample	*scarce*
Distressed country	*constrained*	region I	region II
	unconstrained	region IV	region III

Table 1: Regions for equilibrium in state (i, s) or (s, i).

These four types of equilibria are displayed in Figure 3.[6] As represented in table 1, the regions can be categorized according to (a) whether global liquidity is ample or scarce, and (b) whether the distressed country's collateral constraint is slack or binds. When global liquidity is ample (regions I and IV), the equilibrium interest rate is low: $R^{is} = 1$. In region I, the distressed country is constrained, and there is partial liquidation ($\theta_B^{is} = (\kappa r k_B + \ell_B)/k_B < 1$). In region IV, the distressed country is unconstrained, and borrowing is high enough to allow for full continuation and avoid partial liquidation ($\theta_B^{is} = 1$). When, on the other hand, global liquidity is scarce (regions II and III), the equilibrium interest rate rises: $R^{is} > 1$. In these regions, all the global liquidity is used to shore up the distressed country's assets, but the aggregate shortage of date 1 resources results in partial liquidation ($\theta_B^{is} = (\ell_A + \ell_B)/k_B < 1$). In region II, the distressed country is constrained and can only pledge to offer to lenders a return $R^{is} = \kappa r k_B/\ell_A$, lower than the social marginal return Δ. In region III, on the other hand, the distressed country is unconstrained and the lender country can be compensated at the social marginal return of liquidity in consumption goods terms, $R^{is} = \Delta$. As is common in models with borrowing constraints (i.e. Bernanke and Gertler 1989, Kiyotaki and Moore 1997), a wedge between the internal and external rate of return on investment arises in regions I and II.

The relevant region for equilibrium on the date 1 spot market in states (i, s) and (s, i) depends on date 0 choices in the two countries. The next section provides conditions on parameters under which date 0 choices in a competitive equilibrium lead to particular regions. The focus of the analysis will henceforth be on parameter configurations which lead the competitive equilibrium to be in region II.

2.2.2 Decentralized equilibrium

At date 0, an agent in country j takes the schedule of date 1 interest rates R^ω as given and solves

$$\max_{k_j, \ell_j} \sum_{\omega \in \Omega} \pi^\omega V^\omega(k_j, \ell_j) \tag{6}$$

subject to

$$k_j + \ell_j = 1, \tag{7}$$

where the date 1 value function $V^\omega(k_j, \ell_j)$ is equal to $V_i^\omega(k_j, \ell_j)$ if the agent is intact and to $V_s^\omega(k_j, \ell_j)$ if the agent is distressed. The first-order condition

$$\sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial k_j} = \sum_{\omega \in \Omega} \pi^\omega \frac{\partial V^\omega(k_j, \ell_j)}{\partial \ell_j} \tag{8}$$

together with the date 0 budget constraint (7) characterize the agent's optimal choice. A *competitive equilibrium* of the model consists of date 0 decisions $(k_j, \ell_j)_{j \in \{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega \in \Omega, j \in \{A,B\}}$ and prices $(R^\omega)_{\omega \in \Omega}$, such that (a) given prices, the decisions solve the problems

[6] The equilibria are drawn for state (i, s), where country A is intact and country B is distressed, but equilibria in state (s, i) take identical forms, with the subscript A and B interchanged.

in (6), (1) and (2); and (b) markets clear.[7] In what follows, the values of k and ℓ in a symmetric competitive equilibrium are denoted by k^{CE} and ℓ^{CE}.

Assumption 2 guarantees that the probability of a crisis is not large enough to produce a situation where the aggregate amount of liquidity set aside in a symmetric competitive equilibrium, $2\ell^{CE}$, is sufficient to avoid any liquidation in the states of the world where one country is intact and the other is distressed. In other words, assumption 2 ensures that $k^{CE} > 2/3$ in a symmetric competitive equilibrium. This implies that we can focus on situations in which only regions I, II or III in states (i, s) and (s, i) can arise in equilibrium.

To gain further insights into the properties of a competitive equilibrium of the model, it is useful to look at the agents' value function. In state ω, the value function is given by

$$V_i^{\omega}(k_j, \ell_j) = u\Big(Ak_j + R^{\omega}\ell_j\Big) \tag{9}$$

for an intact agent, and by

$$V_s^{\omega}(k_j, \ell_j) = u\Big(rk_j + (\Delta - R^{\omega})\frac{\kappa r k_j}{R^{\omega}} + \Delta\ell_j\Big) \tag{10}$$

for a distressed agent. The terms in (9) are evident to interpret. For an intact agent, illiquid assets yield a return of A, while liquid assets yield a return of R^{ω}, with $1 \leq R^{\omega} \leq \Delta$. The marginal value of the illiquid asset in terms of date 2 consumption (rather than date 2 utility) is higher than that of the liquid asset. The terms in (10) are similarly straightforward. For a distressed agent, each unit of illiquid assets yields a baseline return of r, plus a net return of $\Delta - R^{\omega}$ on the $\kappa r/R^{\omega}$ of external financing raised against collateral, whereas a unit of liquid assets allows the continuation of one unit of illiquid assets, yielding a return of Δ. The marginal value of the liquid asset is higher than that of the illiquid asset.[8] Given concave utility, from the perspective of period 0 the illiquid asset is a bad hedge, while the liquid asset is a good hedge.

It turns out that the equilibrium can be further characterized as falling into one of the aforementioned three regions, depending on the tightness of financial constraints κ. There are thresholds $\underline{\kappa}$ and $\bar{\kappa}$, such that

- for $\kappa < \underline{\kappa}$, the symmetric competitive equilibrium leads to region I, i.e. $\frac{2}{3} < k^{CE} < \frac{1}{1+\kappa r}$,
- for $\underline{\kappa} \leq \kappa \leq \bar{\kappa}$, the symmetric competitive equilibrium leads to region II, i.e. $\frac{1}{1+\kappa r} \leq k^{CE} \leq \frac{\Delta}{\Delta+\kappa r}$,
- for $\kappa > \bar{\kappa}$, the symmetric competitive equilibrium leads to region III, i.e. $\frac{\Delta}{\Delta+\kappa r} < k^{CE} < 1$.

Hence, very tight financial constraints lead to region I, mildly tight constraints lead to region II, and loose constraints lead to region III. The interest rate in states (i, s) and (s, i) is pictured as a function of the tightness of financial constraints in Figure 4.

[7] The continuation scale of an intact agent is by definition set to $\theta_j^{\omega} = 1$.

[8] The marginal value of the illiquid asset for a distressed agent is the highest when $R^{\omega} = 1$, in which case it is given by $r + (\Delta - 1)\kappa r = [(1 - \kappa) + \kappa\Delta]r \leq \Delta r < \Delta$.

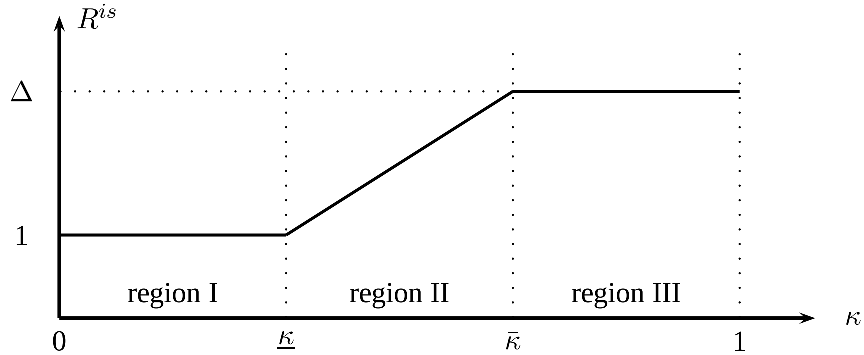

Figure 4: Equilibrium interest rate in state (i, s) as a function of κ, in competitive equilibrium.

For the remainder of the paper, we focus on the case in which the symmetric competitive equilibrium leads to region II, via the following assumption.

Assumption 3 (Tightness of financial constraints)**.** *The tightness of financial constraints κ satisfies $\underline{\kappa} < \kappa < \bar{\kappa}$.*

Assumption 3 guarantees that the symmetric competitive equilibrium leads to the interior of region II, i.e. that $\frac{1}{1+\kappa r} < k^{CE} < \frac{\Delta}{\Delta+\kappa r}$. The nature of the date 1 spot market equilibrium in region II provides a stylized description of actual liquidity crises in a globally integrated financial system in two key respects: (a) the aggregate shortage of liquidity results in a spike in the cost of borrowing, and (b) the pervasiveness of financial constraints causes a wedge between the internal and external marginal value of funds for distressed entities. Importantly, in this region, the price of liquidity (i.e. the interest rate) is a decreasing function of the amount of liquidity set aside ex-ante by lenders, and an increasing function of the amount of illiquid collateral owned by borrowers. Equilibrium in this region therefore captures the key intuition that the price of liquidity in crises decreases with the supply of it and increases with the demand for it. A lower ex-ante illiquid investment scale decreases the interest rate in crises, which benefits distressed borrowers more than it hurts intact lenders at the margin. Due to the incompleteness of markets, this pecuniary externality causes a market failure: the date 0 choices in a competitive equilibrium are not constrained efficient as perturbating these allocations locally has first-order welfare effects via shifts in interest rates. The failure of the competitive equilibrium to be constrained efficient motivates the analysis of prudential regulation from a second best perspective.

3 Efficiency and planning problems

In assessing the welfare performance of competitive equilibria in incomplete markets economies, on is generally interested in whether the market system allocates resources efficiently given the set of markets operating (see Stiglitz 1982). This section analyzes alternative allocation mechanisms and compares their welfare properties with those of the decentralized equilibrium described in section 2.2. Section 3.1 starts by providing a brief description of the frictionless first-best allocation benchmark. Sections 3.2 and 3.3 then characterize the allocations resulting from global and national planners making investment decisions subject to the same set of enforcement and informational frictions as private agents. These allocation mechanisms are interpreted as regulated equilibria, as in Allen and Gale (2004).

3.1 First-best allocation

For the sake for notational simplicity, the first-best allocation is derived as the solution to a planning problem. The first welfare theorem guarantees that this allocation coincides with the one obtained in a competitive equilibrium under complete markets.

Given symmetry, an unconstrained social planner maximizes the equally weighed sum of the agents' expected utility subject to resource constraints only. His problem is to solve

$$\max_{(k_j,\ell_j,c_j^\omega)_{j\in\{A,B\},\omega\in\Omega}} \sum_{\omega\in\Omega} \pi^\omega[u(c_A^\omega)+u(c_B^\omega)] \tag{11}$$

subject to

$$\begin{aligned} k_A+k_B+\ell_A+\ell_B &= 2 && (12)\\ c_A^\omega+c_B^\omega &= Y_2^\omega(k_A,k_B,\ell_A,\ell_B) \qquad \text{for} \qquad \omega\in\Omega && (13) \end{aligned}$$

where $Y_2^\omega(k_A,k_B,\ell_A,\ell_B)$ denotes aggregate date 2 resources:

$$\begin{aligned} Y_2^{ii}(k_A,k_B,\ell_A,\ell_B) &= A(k_A+k_B)+\ell_A+\ell_B\\ Y_2^{is}(k_A,k_B,\ell_A,\ell_B) &= Ak_A+rk_B+\Delta(\ell_A+\ell_B)\\ Y_2^{si}(k_A,k_B,\ell_A,\ell_B) &= rk_A+Ak_B+\Delta(\ell_A+\ell_B)\\ Y_2^{ss}(k_A,k_B,\ell_A,\ell_B) &= r(k_A+k_B)+\Delta(\ell_A+\ell_B) \end{aligned}$$

(12) is the date 0 worldwide resource constraint, and (13) is the date 2 worldwide resource constraints in state ω, for an efficient use of date 1 goods (i.e. liquidity). In state (i,i), all date 1 goods are invested in the storage technology and yield a date 2 return of 1. In the three other states, all date 1 goods are used to shore up distressed long-term assets, yielding a date 2 return of $\Delta>1$.

The first-order conditions for c_A^ω and c_B^ω imply that marginal utilities are equalized across countries in each state ω. It follows that consumption is also always equalized across countries. This naturally implies that the marginal rate of substitution of consumption between different states of nature are also equalized across agents. This equalization is characteristic of exchange efficiency.

The first-order conditions for k_A, k_B, ℓ_A and ℓ_B imply that the planner's investment decisions satisfy $k_A = k_B = k^*$, $\ell_A = \ell_B = \ell^* = 1 - k^*$, and

$$\sum_{\omega\in\Omega} \pi^\omega \frac{\partial V^{*\omega}(k^*, k^*, 1-k^*, 1-k^*)}{\partial k_j} = \sum_{\omega\in\Omega} \pi^\omega \frac{\partial V^{*\omega}(k^*, k^*, 1-k^*, 1-k^*)}{\partial \ell_j} \qquad \text{for} \quad j \in \{A, B\}, \tag{14}$$

where $V^{*\omega}(k_A, k_B, \ell_A, \ell_B) = u\Big(\frac{Y_2^\omega(k_A,k_B,\ell_A,\ell_B)}{2}\Big)$. (14) says that the planner's investment plan results in an equalization of the discounted expected returns on the two assets, with weights corresponding to the representative agent's stochastic discount factor. This equalization is characteristic of production efficiency in an economy with complete markets.

3.2 Constrained global planner

Given the presumption of the failure of the first welfare theorem, it is natural to ask how a constrained social planner would want to regulate date 0 investment decisions. To this end, we start by considering a global planner who maximizes the sum of agents' expected utility in the two countries, makes date 0 decisions about k and ℓ instead of private agents in both countries, and lets the spot market operate competitively at date 1. Importantly, the planner is assumed to be subject to the same set of informational and enforcement constraints as the private sector.

Since agents are ex-ante homogenous, the planner is assumed to assign equal weights to agents in the two countries when making date 0 choices. The planner's date 1 value function $\tilde{V}^\omega(k_A, k_B, \ell_A, \ell_B)$ is given by the sum of the respective expressions in (9) and (10) in which the equilibrium interest rate has been substituted in. When the interest rate is not an explicit function of date 0 choices, as in states (i,i), (s,s), and $(i,s)/(s,i)$ in region I, III and IV, $\tilde{V}^\omega(k_A, k_B, \ell_A, \ell_B)$ coincides with the sum of the value functions perceived by the agents in the two countries. But when the interest rate depends explicitly on date 0 choices, as in states $(i,s)/(s,i)$ in region II, the planner's value functions are given by

$$\tilde{V}^{is}(k_A, k_B, \ell_A, \ell_B) = u\Big(Ak_A + \kappa r k_B\Big) + u\Big((1-\kappa) r k_B + \Delta(\ell_A + \ell_B)\Big), \tag{15}$$

$$\tilde{V}^{si}(k_A, k_B, \ell_A, \ell_B) = u\Big((1-\kappa) r k_A + \Delta(\ell_B + \ell_A)\Big) + u\Big(Ak_B + \kappa r k_A\Big). \tag{16}$$

Comparing the expressions in (15-16) with those in (9-10), it is apparent that the global planner's marginal valuation of the two assets does not coincide with the private marginal valuation in states (i,s) and (s,i). This result is formalized in the following lemma.

Lemma 1 (Differences in asset valuations between global planner and private agents)**.** *In region II, the global planner values*

1. *the intact country's liquid assets more highly than private agents in the state of nature where the other country is distressed, and*

2. *the distressed country's illiquid assets less highly than private agents in the state of nature where the other country is intact.*

Proof. See appendix. □

The two results in lemma 1 reflect the fact that the global planner internalizes the effect of the two countries' asset positions on the interest rate in crises, while private agents take this price as given. The undervaluation of liquid assets by private agents (part 1 of the lemma) can be traced back to two separate effects. First, the social return in terms of date 2 goods of a marginal unit of liquid assets in the hands of an intact agent is Δ, but because of binding financial constraints, intact agents only earn a marginal return of $R^{is} < \Delta$ in equilibrium. Second, given concave utility, distressed agents value date 2 resource more highly than intact agents, so the decrease in the interest rate brought about by an additional marginal unit of liquidity supply benefits more the distressed borrower than it hurts the intact lender. The overvaluation of illiquid assets by distressed agents (part 2 of the lemma) relies solely on this latter wealth redistributive effect. At the margin, a lower stock of illiquid assets for the distressed country, by reducing the amount of collateral available for loans, reduces the demand for loans and therefore reduces the interest rate. This reduction in the interest rate transfers wealth from intact lenders to distressed borrowers, and results in a net gain in social welfare.

The differential asset valuation result emphasized in lemma 1 leads the planner to make date 0 investment choices that generally differ from those obtained in a competitive equilibrium. The global planner solves

$$\max_{(k_j,\ell_j)_{j\in\{A,B\}}} \sum_{\omega\in\Omega} \pi^\omega \tilde{V}^\omega(k_A, k_B, \ell_A, \ell_B) \tag{17}$$

subject to

$$k_j + \ell_j = 1 \qquad \text{for} \quad j \in \{A, B\}. \tag{18}$$

In other words, the planner makes date 0 choices while anticipating the effect of its decisions on the determination of the spot market equilibrium at date 1. A *globally regulated equilibrium* consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices, the private sector's decisions solve the problems in (1) and (2); (b) the global planner's decisions solve the problem in (17); and (c) markets clear. For future reference, the level of k and ℓ chosen by a global planner in a symmetric optimal plan are denoted by $\tilde{k}$ and $\tilde{\ell}$.

How does $\tilde{k}$ relate to k^{CE}? As noted above, for $0 \le k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} < k \le 1$, the planner's objective coincides with the private agents' objectives, since in these regions (I, III and IV), the interest rate is not a function of date 0 choices locally. Furthermore, under assumptions 1, 2 and 3, the investment choice k^{CE} in a competitive equilibrium falls in the interior of the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$ while the private agents' objective is monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. Since over these latter two intervals, the planner's and private agents' objectives are the same, it has to be that the planner's objective is also monotonically increasing over $\left[0, \frac{1}{1+\kappa r}\right)$ and monotonically decreasing over $\left(\frac{\Delta}{\Delta+\kappa r}, 1\right]$. The global planner's optimal investment choice $\tilde{k}$ therefore has to fall in the interval $[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}]$ (region II). $\tilde{k}$ thus necessarily satisfies

$$\sum_{\omega\in\Omega}\pi^{\omega}\frac{\partial\tilde{V}^{\omega}(\tilde{k},\tilde{k},1-\tilde{k},1-\tilde{k})}{\partial k_j}-\sum_{\omega\in\Omega}\pi^{\omega}\frac{\partial\tilde{V}^{\omega}(\tilde{k},\tilde{k},1-\tilde{k},1-\tilde{k})}{\partial \ell_j}\lesseqgtr 0 \qquad \text{for} \quad j\in\{A,B\}, \tag{19}$$

with "$\leq$" if $\tilde{k}=\frac{1}{1+\kappa r}$, with "$=$" if $\frac{1}{1+\kappa r}<\tilde{k}<\frac{1}{1+\kappa r}$ and with "$\geq$" if $\tilde{k}=\frac{\Delta}{\Delta+\kappa r}$.

Proposition 1 (Excessive illiquidity in competitive equilibrium)**.** *A global planner chooses a more liquid and less risky investment portfolio than private agents in the competitive equilibrium, i.e.* $\tilde{k}<k^{CE}$ *and* $\ell^{CE}<\tilde{\ell}$*.*

Proof. See appendix. □

Proposition 1 establishes the constrained inefficiency of decentralized investment decisions in the model, and provides a characterization of the constrained efficient allocation. At date 1, the global planner's and private agents' valuations of the liquid asset coincide in all states of nature, except in the one where agents lend to distressed foreigners and the planner's valuation is strictly higher. Similarly, the planner's and private agents' valuations of the illiquid asset coincide in all states, except in the one where agents borrow from intact foreigners and the planner's valuation is strictly lower. These wedges between the private and social valuations of the respective assets naturally lead the global planner to invest more in the liquid asset and less in the illiquid asset at date 0.

Since agents in both countries are ex-ante identical, the welfare metric is unambiguously given by a representative agent's expected utility at date 0. This criterion corresponds to the global planner's objective, re-scaled by 1/2. Since the planner's objective is strictly decreasing in k for $k\geq\tilde{k}$, welfare is strictly higher under global regulation than in the decentralized equilibrium.[9] As developed further in section 3.4, the global planner recognizes that a more liquid investment portfolio at date 0 brings about a redistribution of wealth from intact lenders to distressed borrowers in states (i,s) and (s,i).

Global regulation makes financial crises less severe in two respects. First, there is always less liquidation during a crisis with global regulation than in the laissez-faire case. In states of nature where one country is distressed and the other is intact, liquidation is given by $1-\theta^{\omega}=1-2(1-k)/k$, while in the state where both countries are distressed, liquidation is given by $1-\theta^{\omega}=1-(1-k)/k$. In both cases, $\tilde{k}<k^{CE}$ implies that there are less illiquid investment to shore up in a crisis and more available liquid resources to do so under global regulation than in a decentralized equilibrium. Second, global regulation results in less pronounced interest rate spikes when one country is hit and the other is not, since $R^{is}=R^{si}=\kappa rk/(1-k)$. The demand for funds is smaller, and the supply of funds is larger, resulting in a milder increase in the price of liquidity in a crisis.

[9]The fact that the planner's objective is strictly decreasing in k for $\tilde{k}\leq k\leq\frac{\Delta}{\Delta+\kappa r}$ follows from the planner's second-order condition.

3.3 Constrained national planners

In order to understand the source of tensions that can arise in an environment where regulations are set independently in each country, we now consider the case of national planners who make date 0 decisions in their respective countries and let the spot market operate competitively at date 1. The assumption that planners are subject to the same informational and enforcement frictions than private agents is maintained.

The national planners are assumed to maximize the expected utility of domestic agents when making date 0 choices. Country j's planner date 1 value function $\hat{V}^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j})$ is given by the expression in (9) or (10) in which the equilibrium interest rate has been substituted in. As was the case with the global planner, when the interest rate is not an explicit function of date 0 investment choices, the national planners' value function coincides with the private agents' value function[10]. But in states $(i, s)/(s, i)$ in region II, where the interest rate is a function of date 0 choices, the planners' value functions are given by

$$\hat{V}_i^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big(Ak_j + \kappa r k_{-j}\Big), \tag{20}$$

$$\hat{V}_s^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j}) = u\Big((1-\kappa) r k_j + \Delta(\ell_{-j} + \ell_j)\Big). \tag{21}$$

A comparison of the expressions in (20-21) with those in (15-16) and (9-10) reveals that the national planner's valuation of the two assets neither coincides with the global planner's valuation, nor with the private valuation in states (i, s) and (s, i). These wedges in valuations are formalized in the following two lemmas.

Lemma 2 (Undervaluation of assets by national planners vs. global planner)**.** *A national planner values*

1. *its intact agents' liquid assets less highly than the global planner in states of nature where the other country is distressed, and*

2. *its distressed agents' illiquid assets less highly than the global planner in states of nature where the other country is intact.*

Proof. See appendix. □

The undervaluation of an intact agent's liquid assets by a national planner vs. the global planner (part 1 of the lemma) is a consequence of the public goods property of international liquidity in a crisis. When a country is distressed and the other one is intact, the marginal value of either country's liquidity holding accrues *entirely* to the distressed country. At the margin, the value of an additional unit of liquid assets for the intact country is thus literally zero. This somewhat surprising result follows from the fact that an extra unit of liquid assets decreases the interest rate at which the intact country is lending in a way that makes the total revenues from lending abroad

[10] This is again the case in states (i, i), (s, s), and $(i, s)/(s, i)$ in regions I, III and IV

insensitive to the intact country's holdings of liquid assets (at least locally):

$$\frac{\mathrm{d}(R^{is}\ell_A)}{\mathrm{d}\ell_A} = \underbrace{\frac{\kappa r k_B}{\ell_A}}_{R^{is}} - \ell_A \underbrace{\frac{\kappa r k_B}{\ell_A^2}}_{\partial R^{is}/\partial \ell_A} = 0.$$

The global planner, on the other hand, values liquid asset holdings by both countries equally and at their full social returns.

The undervaluation of distressed agents' illiquid assets by a national planner vs. the global planner (part 2 of the lemma) also reflects the fact that the distressed and intact countries share the social value of an additional unit of collateral, with the share depending on the degree of pledgeability κ. The distressed country's national planner only captures a share $1-\kappa$ of the marginal value of the illiquid asset in terms of date 2 goods, and thus naturally undervalues the illiquid asset.

The undervaluation results for national planners relative to the global planner also hold vis-a-vis private agents.

Lemma 3 (Undervaluation of assets by national planners vs private agents)**.** *A national planner values*

1. *its intact agents' liquid assets less highly than the agents themselves in states of nature where the other country is distressed, and*

2. *its distressed agents' illiquid assets less highly than the agents themselves in states of nature where the other country is intact.*

Proof. See appendix. □

The undervaluation of intact agents' liquid assets by national planners (part 1 of the lemma) reflects the fact that national planners internalize the drop in the interest rate caused by a marginally larger holding of liquid assets, while private agents take interest rates as given. The undervaluation of distressed agents' illiquid assets by national planner (part 2 of the lemma), on the other hand, follows directly from the fact that distressed private agents overvalue illiquid assets relative to the global planner, while national planners undervalue illiquid assets relative to the global planner.

Given the wedges in asset valuations between the national planners and the global planner, it is clear that regulations chosen at the national level will generally not coincide with the constrained optimal allocation. Country j's national planner solves

$$\max_{k_j,\ell_j} \sum_{\omega\in\Omega} \pi^{\omega} \hat{V}^{\omega}(k_j, k_{-j}, \ell_j, \ell_{-j}) \tag{22}$$

subject to (7). National planners make date 0 choices while anticipating the effect of their decisions on the determination of the spot market equilibrium at date 1, and taking the action of the other country's national planner as given. A *nationally regulated equilibrium* (NRE) consists of date 0 decisions $(k_j, \ell_j)_{j\in\{A,B\}}$, date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and date 0 decisions, the private sector's date 1 decisions solve the problems in (1) and (2); (b) given (k_{-j}, ℓ_{-j}), (k_j, ℓ_j) solves the problem in (22); and (c) markets clear. The level of k and ℓ chosen by national planners in a symmetric nationally regulated equilibrium are denoted by $\hat{k}$ and $\hat{\ell}$.

How does $\hat{k}$ relate to k^{CE} and $\tilde{k}$? We observe that under symmetric choices, for $0 \le k < \frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r} \le k \le 1$, the national planners' objectives coincide with both the private agents' and the global planner's objective since in these regions the interest rate does not depend on date 0 choices locally. An argument analogous to that used in section 3.2 implies that the national planners' investment choices in a symmetric nationally regulated equilibrium $\hat{k}$ have to fall in the interval $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$ (i.e. in region II). A necessary condition for a symmetric nationally regulated equilibrium is therefore given by

$$\sum_{\omega\in\Omega} \pi^\omega \frac{\partial \hat{V}^\omega(\hat{k}, \hat{k}, 1-\hat{k}, 1-\hat{k})}{\partial k_j} - \sum_{\omega\in\Omega} \pi^\omega \frac{\partial \hat{V}^\omega(\hat{k}, \hat{k}, 1-\hat{k}, 1-\hat{k})}{\partial \ell_j} \lesseqqgtr 0 \qquad \text{for} \quad j \in \{A, B\}, \tag{23}$$

with "$\le$" if $\hat{k} = \frac{1}{1+\kappa r}$, with "$=$" if $\frac{1}{1+\kappa r} < \hat{k} < \frac{1}{1+\kappa r}$ and with "$\ge$" if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$.

Proposition 2 (Excessive illiquidity of NRE relative to GRE)**.** *National planners choose a weakly less liquid and more risky investment portfolio than a global planner, i.e.* $\hat{\ell} \le \tilde{\ell}$ *and* $\hat{k} \ge \tilde{k}$*. Furthermore, if* $\hat{k} > \frac{1}{1+\kappa r}$*, then national planners choose a strictly less liquid and more risky investment portfolio than a global planner, i.e.* $\hat{\ell} < \tilde{\ell}$ *and* $\hat{k} > \tilde{k}$*.*

Proof. See appendix. □

Lemma 2 had established that national planners (a) undervalue liquid assets when their country is intact and the foreign country is distressed, and (b) undervalue illiquid assets when their country is distressed and the foreign country is intact. Proposition 2 states that unless both equilibria result in a (left) corner solution for the date 0 investment choice (i.e. when $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$), the undervaluation of liquid assets by national planners dominates the undervaluation of illiquid assets, so that wedges in ex-post valuations of assets result in an excessive illiquidity of investment by national planners relative to the constrained efficient allocation. Failing to coordinate macro-prudential policy results in an insufficient amount of regulation. In other words, the public goods property of international liquidity during a crisis translates into a public goods property of prudential regulation. Since the global planner's objective is strictly decreasing in k for $k \ge \tilde{k}$, this insufficient amount of regulation results in a weakly lower welfare than in the constrained efficient allocation (strictly lower if $\hat{k} > \frac{1}{1+\kappa r}$). Insufficient provision of regulation also results in more liquidation and higher interest rates during financial crises.

Without imposing additional structure on the primitives, the relationship between $\hat{k}$ and k^{CE} is ambiguous. It is therefore not a priori clear whether national planners want to hoard more or less liquidity than private agents in a competitive equilibrium. It turns out that by putting more structure on preferences, one can obtain the result that national planners want to hoard less liquidity than private agents, as illustrated in the following proposition.

Proposition 3 (Excessive illiquidity of NRE relative to CE)**.** *When utility is logarithmic, national planners choose a less liquid and more risky investment portfolio than private agents in the competitive equilibrium, i.e.* $\hat{\ell} < \ell^{CE}$ *and* $\hat{k} > k^{CE}$.

Proof. See appendix. □

Proposition 3 illustrates that the national planners' extremely low valuation of liquidity in the state of nature where its country is intact but the foreign country is distressed can result in more illiquidity and more risk-taking ex-ante than in the laissez-faire benchmark.[11] National regulators may find it optimal to reduce investment in the liquid asset *below* what would be chosen in the free market. This excessive illiquidity results in lower welfare, as measured by the ex-ante expected utility of a representative agent, than both the constrained efficient allocation (achieved by global regulation) *and* the decentralized equilibrium. It also results in more severe financial crises in the form of more liquidation and larger interest rate spikes.

The underprovision of regulation in the absence of international coordination can be viewed as arising from the public goods property international liquidity in crises. Because global liquidity benefits mainly distressed countries during crises, it is attractive to take advantage of foreign liquidity provision when one is distressed, but not to provide liquidity to distressed foreigners when one is intact. Since hoarding liquidity ex-ante is costly in terms of forgone higher expected returns on long-term projects, in a non-cooperative equilibrium countries choose to contribute too little to the pool of international liquidity, and attempt to free-ride on the foreign contribution. This results in a form of beggar-thy-neighbor policy in the area of financial regulation.

Viewed through the lens of the literature on international policy coordination, the underprovision of regulation result can be interpreted as arising from the attempt by national regulators to make use of monopoly and monopsony power in the market for international liquidity during crises. Regulators recognize that less liquidity hoarding ex-ante results in liquidity supply being scarcer in the state of nature where their country will be lending to distressed foreigners. This scarcity is associated with a higher interest rate, and thus brings about a shift in surplus from foreign borrowers to domestic lenders. Regulators also recognize that less illiquid investment ex-ante results in liquidity demand being smaller in the state of nature where their country will be borrowing from

[11] Proposition 3 can be generalized for the class of CRRA utility when the coefficient of relative risk aversion σ is not too large, i.e. when $\sigma < \bar{\sigma}$, for some $\bar{\sigma} > 1$. Intuitively, the relationship between $\hat{k}$ and k^{CE} depends upon two counteracting effects. By hoarding less liquidity than private agents in a competitive equilibrium, national planners benefit from a higher interest rate when their country is intact and lends abroad, but they suffer from the same higher interest rate when their country is distressed and borrows from abroad. The interest rate is more sensitive to liquidity supply than to liquidity demand ($|\partial R^{is}/\partial \ell_A| > \partial R^{is}/\partial k_A$), but, due to concave utility, goods are more valuable when a country is distressed and borrowing than when it is intact and lending. When risk-aversion is low, the utility benefits associated with the first effect dominates: monopoly rents in the market for liquidity (e.g. in state (i, s) for country A) are more important in utility terms than monopsony rents (in state (s, i)). National planners therefore find it optimal to set aside less liquidity than private agents. When risk-aversion is high, the utility costs associated with the second effect dominate because goods are a lot more valuable in the state where the country is distressed and borrowing. National planners then find it optimal to set aside more liquidity than private agents.

intact foreigners. This smaller demand is associated to a lower interest rate, and thus to a shift in surplus from foreign lenders to domestic borrowers. Under the assumption of proposition 3, it turns out that the larger sensitivity of the interest rate to liquidity supply by intact agents than to liquidity demand by distressed agents results in an underinvestment in liquid assets ex-ante by national planners relative to the laissez-faire benchmark.

Finally, the underprovision of regulation result can be seen as arising from a variation of the *hold-up problem* (see Grout 1984 and Tirole 1986). The hold-up problem occurs "when part of the return on an agent's relationship-specific investment is ex-post expropriable by his trading partner" (Che and Sakovics 2008). In the game between national regulators, the allocation of ex-post surplus is achieved via a competitive spot market rather than via bargaining. But the fact is that this allocation results in the distressed country expropriating part of the return on the ex-ante investment in liquid assets by its trading partner (i.e. the intact country). As in the classical hold-up problem, this results in an underinvestment in liquid assets relative to the cooperative solution.

3.4 Exchange efficiency and production efficiency

This section offers a conceptual description of the trade-offs faced by a constrained planner (corresponding to the global planner) when choosing how to set regulations in the present model. In particular, it shows how the planner's decision to regulate can be cast into the management of wedges describing deviations from optimality conditions for exchange and production efficiency. This discussion clarifies why planners controlling only a subset of the economy (national planners) are doomed to miss the goal of regulation in such a framework.

It is well known that incomplete markets generally result in a failure of exchange efficiency. This failure takes the form of wedges between the marginal rates of substitution (MRS) between two goods across agents. In the model of section 2, the relevant MRS is the one between the consumption good in state (i, s) and the same consumption good in state (s, i):

$$MRS_j \equiv \frac{\pi^{is} u'(c_j^{is})}{\pi^{si} u'(c_j^{si})}$$

If markets were complete, agents in the two countries would trade securities contingent on these two states and the MRS would be equalized in a competitive equilibrium. When markets are incomplete, a wedge between the MRS, indicating a failure of exchange efficiency, generally persists in equilibrium. For an arbitrary symmetric date 0 investment choice $(k, 1-k)$, this exchange wedge is given by

$$\tau_e(k) \equiv 1 - \frac{MRS_A(k)}{MRS_B(k)}. \tag{24}$$

It can be shown that $\tau_e(k)$ is monotonically increasing over $[0, 1]$, with $\tau_e(0) = 0$ and $\tau_e(1) = 1 - [u'(A)/u'(r)]^2$. In other words, the higher the investment in illiquid project, the more serious the failure of exchange efficiency in this economy. Intuitively, more investment in illiquid projects leads to more risk, and since markets are incomplete, this higher risk has to be borne by agents. Under assumptions 1, 2 and 3, the symmetric competitive equilibrium falls in region II (i.e. $\frac{1}{1+\kappa r} < k^{CE} < \frac{\Delta}{\Delta+\kappa r}$), in which the wedge is given by

$$\tau_e(k) = 1 - \left[\frac{u'\Big((A+\kappa r)k\Big)}{u'\Big([(1-\kappa)r - 2\Delta]k + 2\Delta\Big)} \right]^2 .$$

Given that $\tau_e'(k) > 0$, by limiting the investment scale via regulation to $\tilde{k} < k^{CE}$, the global planner reduces the size of the wedge relative to the competitive equilibrium, and thereby reduces the severity of exchange inefficiency in the economy. Why doesn't the planner choose to reduce exchange inefficiency further by lowering the investment scale below $\tilde{k}$? The answer lies in the trade-off between exchange inefficiency and production inefficiency faced by the planner. The production wedge can be defined as

$$\tau_p(k) \equiv 1 - \frac{E[R_k^\omega(k)u'(c_j^\omega(k))]}{E[R_\ell^\omega(k)u'(c_j^\omega(k))]}, \tag{25}$$

where R_k^ω and R_ℓ^ω denote the returns on the illiquid and liquid assets, respectively, and $c^\omega(k)$ denotes a representative agent's equilibrium consumption in state ω, for a given symmetric investment choice of k at date 0.[12] The production wedge τ_p is zero at the competitive equilibrium investment decision k^{CE} and non-zero elsewhere. When lowering the investment scale down to $\tilde{k}$, a global planner trades off an improvement in exchange efficiency with a deterioration in production efficiency. Optimal regulation is then the result of the optimal management of these two wedges by the planner.

The above discussion makes it clear that national planners who have pricing power by definition *cannot* be expected to set regulation optimally in an environment where the motivation to regulate originates from a market incompleteness. National planners do not care about overall efficiency, but rather attempt to manipulate prices in a way that results in a shift in surplus in favor of the agents they represent. According to proposition 3, having them in charge of regulation can even result in a deterioration of both exchange efficiency - $\tau_e(k)$ is monotonically increasing so $\tau_e(k^{CE}) < \tau_e(\hat{k})$ - and production efficiency - $\tau_p(k)$ is zero only at k^{CE} - vis-a-vis the competitive equilibrium benchmark.

[12] For $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, the expectations in (25) are given by

$$E[R_k^\omega(k)u'(c_j^\omega(k))] = \pi^{ii}Au'\Big((A-1)k+1\Big) + \pi^{is}Au'\Big((A+\kappa r)k\Big) + \pi^{si}\left[r + \Big(\Delta - \frac{\kappa r k}{1-k}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-k}}\right]u'\Big([(1-\kappa)r - 2\Delta]k + 2\Delta\Big) + \pi^{ss}ru'\Big((r-\Delta)k + \Delta\Big)$$

and

$$E[R_\ell^\omega(k)u'(c_j^\omega(k))] = \pi^{ii}u'\Big((A-1)k+1\Big) + \pi^{is}\frac{\kappa r k}{1-k}u'\Big((A+\kappa r)k\Big) + \pi^{si}\Delta u'\Big([(1-\kappa)r - 2\Delta]k + 2\Delta\Big) + \pi^{ss}\Delta u'\Big((r-\Delta)k + \Delta\Big).$$

4 International spillovers

The model developed in section 2 can also be used for studying the effects of changes in macro-prudential policies across borders. Section 4.1 analyzes the impact of changes in macro-prudential policy in one country on risk-taking by market participants abroad. Section 4.2 looks at how changes in regulation in one country affect the incentive to regulate in the other country. Section 4.3 studies the welfare effects of a unilateral introduction of regulation by one of the two countries.

4.1 Regulatory spillovers

How do market participants react to changes in macro-prudential policy abroad? To answer this question, we consider a version of the model of section 2 in which the date 0 choices are set exogenously in country B and made optimally by private agents in country A. A *competitive equilibrium with exogenous regulations abroad* consists of date 0 decisions (k_A, ℓ_A), date 1 decisions $(d_j^\omega, \theta_j^\omega)_{\omega\in\Omega, j\in\{A,B\}}$ and prices $(R^\omega)_{\omega\in\Omega}$, such that (a) given prices and country B regulations (k_B, ℓ_B), the decisions solve the problem in (6) for country A, and the problems in (1) and (2) for both countries; and (b) markets clear.

To understand the relevant transmission channels of prudential policy across countries, it is useful to denote the aggregate date 0 investment choices in country j by (K_j, L_j). While K_B and L_B are set exogenously by country B's regulator, K_A and L_A result from the optimal choices of private agents in country A. When taking decisions atomistically at date 0, private agents correctly forecast the function mapping aggregate investment choices into interest rates in the various states of nature at date 1, but they take aggregate investment decisions in both countries as given. In the neighborhood of the competitive equilibrium, the pricing functions are given by $R^{ii} = 1$, $R^{ss} = \Delta$,

$$R^{is} = \frac{\kappa r K_B}{L_A}, \qquad \text{and} \qquad R^{si} = \frac{\kappa r K_A}{L_B}. \tag{26}$$

Tighter regulations in country B are captured by a marginal decrease in K_B and a corresponding marginal increase in L_B. The following proposition establishes the direction of the effect of tighter regulations in country B on private agents' investment choice in country A.

Proposition 4 (Regulatory spillovers)**.** *In the neighborhood of a symmetric competitive equilibrium, tighter regulations in country B induce private agents in country A to choose a less liquid and more risky investment portfolio.*

Proof. See appendix. ∎

Tighter regulations in country B do not affect private agents' payoffs in country A in states (i,i) and (s,s). However, they result in lower interest rates in states (i,s) and (s,i). In state (i,s), the tighter regulations lead to a smaller demand for funds by country B's distressed agents, which pushes down the interest rate at which country A's intact agents lend. As can be seen from expression (1), this lower interest rate reduces the return on the liquid asset for country A's agents in that state. In state (s,i), the tighter regulations increase the supply of liquidity, which lowers the interest rate at which country A's distressed agents borrow. As is clear from expression (2), a lower interest rate in this state increases the return on the illiquid asset for country A's agents by increasing the wedge between the internal and external value of funds. Hence, tighter regulations in country B decrease the return on the liquid asset and increase the return on the illiquid asset for private agents in country A. This naturally leads these agents to reallocate their investment portfolio towards more illiquid assets.

4.2 Spillovers in incentives to regulate

How are a regulator's incentives affected by a change in macro-prudential policy abroad? This section shows that the interaction between national regulators in the model of section 2 can be understood in terms of the strategic substitutability concept of Bulow, Geanakoplos, and Klemperer (1985). In light of the result in section 4.1 that tighter regulations abroad induce more risk-taking domestically, one could a priori expect that macro-prudential policies are strategic complements. In fact, the model delivers precisely the opposite result, as stated in the following proposition.

Proposition 5 (Strategic substitutabilities in national regulations)**.** *In the neighborhood of a symmetric competitive equilibrium, national regulations are strategic substitutes.*

Proof. See appendix. ☐

The intuition for this result is that in the neighborhood of the symmetric competitive equilibrium, the national planners' payoff functions only depend on the other country's investment choices in the states of nature where there is cross-border borrowing and lending, i.e. in states (i,s) and (s,i). As can be seen from (20) and (21), a marginal increase in the tightness of regulations in country $-j$, in the form of a marginal decrease in k_{-j} and a corresponding marginal increase in ℓ_{-j}, has the following effects:

- It decreases the interest rate payment $\kappa r k_{-j}$ which country j receives from country $-j$ in the state of nature where intact agents in country j lend to distressed agents in country $-j$. This leads to an increase in the marginal value of the illiquid asset in that state, $Au'\Big(Ak_j + \kappa r k_{-j}\Big)$.
- It increases the loan size ℓ_{-j} which country j receives from country $-j$ in the state of nature where distressed agents in country j borrow from intact agents in country $-j$. This decreases the marginal value of investing in the liquid asset, $\Delta u'\Big((1-\kappa)r + \Delta(\ell_{-j}+\ell_j)\Big)$, more than it increases the marginal value of investing in the illiquid asset, $(1-\kappa)ru'\Big((1-\kappa)r+\Delta(\ell_{-j}+\ell_j)\Big)$.

The combination of these two effects makes the relative attractiveness of investing into the liquid assets, i.e. of regulating, a decreasing function of the tightness of regulations in country $-j$. In other words, these two effects imply that the national regulators' actions are strategic substitutes.

4.3 Welfare effects of unilateral regulation

The analysis in section 3 implies that, starting from a competitive equilibrium, the introduction of a regulation requiring agents to marginally increase their holdings of liquid assets in both countries simultaneously is unambiguously welfare improving for all agents.[13] One might also be interested in the welfare effects of a unilateral introduction of regulation. In particular, does such an introduction of regulation increase welfare in the regulated country? Does it increase welfare in the unregulated country? These are relevant questions to the extent that their answer determines the incentives of a national regulator to move first in a world where macro-prudential regulations are absent to start with. This section uses the concept of a competitive equilibrium with exogenous regulation abroad defined in section 4.1 to address these issues.

The focus is on the effects of the introduction of a small regulation requiring country B agents to increase their holding of liquid assets by $dL_B = -dK_B > 0$ relative to the symmetric competitive equilibrium level $\ell^{CE} = 1 - k^{CE}$. We consider in turn the welfare effects in the regulated country and in the unregulated country.

Welfare of regulated country

In a competitive equilibrium with exogenous regulations abroad, country A agents make their date 0 investment decisions taking prices and foreign regulation as given. In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the regulated country (country B) is given by

$$\begin{aligned} W(K_B, k_A) &\equiv \sum_{\omega \in \Omega} \pi^{\omega} \hat{V}^{\omega}(K_B, k_A, 1-K_B, 1-k_A) \\ &= \pi^{ii} u\Big(AK_B + (1-K_B)\Big) + \pi^{is} u\Big((1-\kappa) r K_B + \Delta(1 - K_B + 1 - k_A)\Big) \\ &\quad + \pi^{si} u\Big(AK_B + \kappa r k_A\Big) + \pi^{ss} u\Big(rK_B + \Delta(1-K_B)\Big). \end{aligned} \tag{27}$$

Starting from a symmetric competitive equilibrium, the marginal effect on country B's welfare of a tightening of regulation in country B is given by

$$dW = \Big[\frac{\partial W}{\partial K_B} + \frac{\partial W}{\partial k_A}\frac{dk_A^{CE}}{dK_B}\Big] dK_B, \tag{28}$$

where the derivatives are evaluated at $(K_B, k_A) = (k^{CE}, k^{CE})$, and $dK_B < 0$. The first term in the brackets in (28) represents the direct effect of a change in regulation. The second term in the brackets reflects the indirect effect working through the spillovers operating in country A. Proposition 4 established that a tightening of regulation in country B induced a more illiquid date 0 portfolio choice in country A, i.e. that $\frac{dk_A^{CE}}{dK_B} < 0$. The partial derivatives of $W(K_B, k_A)$, evaluated

[13] This follows from the fact that the global planner's objective is strictly decreasing in k for $k \geq \tilde{k}$, while $k^{CE} > \tilde{k}$.

at (k^{CE}, k^{CE}), are given by

$$\begin{aligned}\frac{\partial W}{\partial K_B} &= \pi^{ii}(A-1)u'\Big((A-1)k^{CE}+1\Big) + \pi^{is}[(1-\kappa)r-\Delta]u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big) \\ &\quad \pi^{si}Au'\Big((A+\kappa r)k^{CE}\Big) + \pi^{ss}(r-\Delta)u'\Big((r-\Delta)k^{CE}+\Delta\Big) \end{aligned} \tag{29}$$

and

$$\frac{\partial W}{\partial k_A} = -\pi^{is}\Delta u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big) + \pi^{si}\kappa r u'\Big((A+\kappa r)k^{CE}\Big). \tag{30}$$

Using the first-order condition of the symmetric competitive equilibrium, $g_{II}(k^{CE}) = 0$ ($g_{II}(k)$ is defined in (A.8)), $\frac{\partial W}{\partial K_B}$ can be written as

$$\frac{\partial W}{\partial K_B} = -\pi^{is}\Delta\frac{\kappa r(1-k^{CE})}{\kappa r k^{CE}}u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big) + \pi^{si}\frac{\kappa r k^{CE}}{1-k^{CE}}u'\Big((A+\kappa r)k^{CE}\Big) \tag{31}$$

Without further restriction, it is not possible to determine the sign of $\frac{\partial W}{\partial K_B}$, so the direct effect of the tightening of regulation is ambiguous. The tightening of regulation pushes the interest rate down in the states (i, s) and (s, i) in which there is international borrowing/lending. In state (i, s), this lowering of the interest rate benefits country B's agents who can borrow more cheaply (first term in (31)). In state (s, i), it costs country B's agents who lend at a lower rate (second term in (31)). Whether the costs are smaller or larger than the benefits is a priori ambiguous. The direction of the indirect effect, however, is unambiguous. Since, as shown in the proof of lemma 1, $(A+\kappa)k^{CE} > [(1-\kappa)r - 2\Delta]k^{CE} + 2\Delta$, the concavity of $u(\cdot)$ implies $\frac{\partial W}{\partial k_A} < 0$. For country B's agents, the losses from an increase in the costs of foreign liquidity provision in state (i, s) (the first term in (30)) is only partially offset by the benefits of an increase in the interest rate payment from abroad in state (s, i) (the second term in (30)). The indirect effect $\frac{\partial W}{\partial k_A}\frac{dk_A^{CE}}{dK_B}$ in (28) is therefore strictly positive. A unilateral tightening of regulations in country B induces more risk-taking and illiquidity abroad, and this feeds back negatively into country B.

It turns out that with more structure imposed on the primitives, the model can deliver a situation where both the direct and the indirect effect work in the same direction. This results in an unambiguous overall effect of a tightening of regulation on the regulated country's welfare, as stated in the following proposition.

Proposition 6 (Beggar-thyself unilateral regulation)**.** *Absent initial regulation, when utility is logarithmic, the unilateral introduction of a (small) regulation is welfare reducing for the country introducing the regulation, i.e.* $\frac{dW}{dK_B} > 0$.

Proof. See appendix. □

Under the logarithmic utility assumption, for reasons similar to those underlying the result in proposition 3, the direct effect of a tightening of regulation on the regulated country's welfare is negative. The losses from the extra costs of lending at a lower interest rate in state (s, i) are larger than the benefits from cheaper borrowing in state (i,s) for country B's agents.[14] While introducing regulation simultaneously at home and abroad is welfare improving for both countries, a unilateral introduction of regulation can quite naturally be welfare reducing for the regulated country, because of terms of trade effects and their associated spillovers. This result is reminiscent of that of Corsetti and Pesenti (2001), who find that an unexpected monetary expansion can be welfare reducing for a given country when carried on unilaterally in an environment where, owing to monopolistic distortions in production and nominal rigidities, the same policy would be welfare improving if pursued simultaneously at home and abroad.

Welfare of unregulated country

In the neighborhood of the symmetric competitive equilibrium, the ex-ante welfare of agents in the unregulated country (country A) is given by

$$\begin{aligned}\Pi(k_A, K_B) &\equiv \sum_{\omega\in\Omega} \pi^{\omega}\hat{V}^{\omega}(k_A, K_B, 1-k_A, 1-K_B) \\ &= \pi^{ii}u\Big(Ak_A + (1-k_A)\Big) + \pi^{is}u\Big(Ak_A + \kappa r k_B\Big) \\ &\quad +\pi^{si}u\Big((1-\kappa)rk_A + \Delta(1-k_A+1-k_B)\Big) + \pi^{ss}u\Big(rk_A + \Delta(1-k_A)\Big).\end{aligned} \tag{32}$$

Starting from a symmetric competitive equilibrium, the marginal effect on country A's welfare of a tightening of regulation in country B is given by

$$d\Pi = \Big[\frac{\partial \Pi}{\partial k_A}\frac{dk_A^{CE}}{dK_B} + \frac{\partial \Pi}{\partial K_B}\Big]dK_B, \tag{33}$$

where the derivatives are evaluated at $(k_A, K_B) = (k^{CE}, k^{CE})$. The overall effect of a tightening of regulation on the unregulated country's welfare is again given by the sum of an indirect effect (first term in (33)) and a direct effect (second term in (33)). The partial derivatives of $\Pi(k_A, K_B)$, evaluated at $(k_A, K_B) = (k^{CE}, k^{CE})$, are given by

$$\begin{aligned}\frac{\partial \Pi}{\partial k_A} &= \pi^{ii}(A-1)u'\Big((A-1)k^{CE}+1\Big) + \pi^{is}Au'\Big((A+\kappa r)k^{CE}\Big) \\ &\quad +\pi^{si}[(1-\kappa)r-\Delta]u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big) + \pi^{ss}(r-\Delta)u'\Big((r-\Delta)k^{CE}+\Delta\Big) \\ &= \frac{\partial W}{\partial K_B},\end{aligned}$$

[14] It is worth noting that the logarithmic utility assumption is merely a sufficient condition for direct effect of a change in regulation on the regulated country's welfare to work in the same direction as the indirect effects. As for proposition 3, the proposition can be generalized for CRRA utility when $\sigma < \bar{\sigma}$, for some $\bar{\sigma} > 1$. The result that the overall effect is negative can be found to hold under less restrictive, albeit not easily characterizable, conditions.

and

$$\begin{aligned}\frac{\partial \Pi}{\partial K_B} &= \pi^{is}\kappa r u'\Big((A+\kappa r)k^{CE}\Big) - \pi^{is}\Delta u'\Big([(1-\kappa)r - 2\Delta]k^{CE} + 2\Delta\Big) \\ &= \frac{\partial W}{\partial k_A} < 0.\end{aligned}$$

The analysis is facilitated by the fact that locally, the effect a small change in a country's portfolio on its own welfare is identical for the regulated and for the unregulated country ($\frac{\partial \Pi}{\partial k_A} = \frac{\partial W}{\partial K_B}$), as is the effect of a small change in a country's portfolio on the other country's welfare ($\frac{\partial \Pi}{\partial K_B} = \frac{\partial W}{\partial k_A}$). The direct effect of a tightening of regulation on the unregulated country's welfare is therefore unambiguously positive, while the direction of the indirect effect is a priori ambiguous. Perhaps paradoxically, it is the unregulated agents' response to the introduction of regulation abroad that may make them worse off than in the absence of any regulation. Without this behavioral response to the introduction of regulation abroad, the unregulated country would necessarily be made better off by the decrease in risk-taking happening abroad. As for the regulated country's welfare, putting more structure on primitives can result in the direct and the indirect effects of the introduction of regulation to work in the same direction. This results in an unambiguously positive overall effect of a tightening of regulation on the unregulated country's welfare.

Proposition 7 (Prosper-thy neighbor unilateral regulation)**.** *Absent initial regulation, when utility is logarithmic, the unilateral introduction of a (small) regulation is welfare improving for the unregulated country, i.e.* $\frac{d\Pi}{dK_B} < 0$*.*

Proof. See appendix. □

With logarithmic utility, the unregulated country's response to the introduction of regulation abroad contributes positively to its welfare. Agents in the unregulated country react by decreasing their investment in liquid assets and increasing their investment in illiquid assets. At the margin, the only impact on their welfare works through the marginal increase in the interest rate in states (i, s) and (s, i) resulting from this portfolio reallocation. The benefits from lending abroad at a higher rate in state (i, s) is larger than the cost of paying a higher rate of foreign loans in state (s, i). A unilateral regulation may therefore be *prosper thy-neighbor* via both the direct and indirect effects.

5 An asset market formulation

This section presents a variant of the model of section 2 in which the intermediation of funds during a crisis occurs via an asset market rather than via a credit market. Distressed agents cannot borrow at date 1, but they can sell some of their illiquid assets to intact agents in order to raise funds. When variables are appropriately relabeled, the date 1 equilibrium of this model is isomorphic to the date 1 equilibrium of the credit market model of section 2. All the results of sections 3.2 to 4 derived for the credit market model therefore also apply to the asset market model of the present section.

As in the baseline model of section 2, markets are incomplete. However, instead of being able to share risk indirectly by borrowing and lending on a credit market at date 1, agents are now able to buy and sell the illiquid asset on a spot market at a price q^ω. Intact agents buy x_j^ω, while distressed agents sell $-x_j^\omega$. We assume that sellers cannot sell more than a fraction η of their total capital holdings k_j. This amounts to assuming limited market liquidity for long-term projects, as in Kiyotaki and Moore (2008). We further assume that the long-term project that are traded need to be shored up by distressed agents before being delivered to a buyer. Buyers of illiquid projects at date 1 therefore recover a return of Ax_j^ω at date 2. All the assumptions of section 2.1 pertaining to preferences, technology and uncertainty are maintained.

At date 1, the value of an intact agent in country j is now given by

$$V_i^\omega(k_j, \ell_j) \equiv \max_{0 \leq x_j^\omega \leq \ell_j / q^\omega} u\Big(Ak_j + Ax_j^\omega + \ell_j - q^\omega x_j^\omega\Big) \tag{34}$$

The agent's date 2 consumption in (34) is given by the sum of the return on its initial holding of illiquid projects Ak_j, the return on the newly acquired illiquid projects Ax_j^ω and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega$. Without loss of generality, we assume that intact agents can only buy and not sell assets on the date 1 spot market. Their capacity to buy is limited by their date 1 liquid resources ℓ_j.

As in the model of section 2, the form of the objective in (34) leads to a simple asset demand schedule for intact agents. For $q^\omega < A$, intact agents exhaust their budget constraint and are willing to buy ℓ_j / q^ω units of the long-term asset. For $q^\omega = A$, they are indifferent between buying any amount between 0 and ℓ_j / A. Finally, for $q^\omega > A$, they do not want to buy any long-term assets. The intact agents' asset demand curve is therefore horizontal at $q^\omega = A$ and slopes downward for $q^\omega < A$.

The date 1 value of a distressed agent in country j is given by

$$V_s^\omega(k_j, \ell_j) \equiv \max_{\theta_j^\omega, x_j^\omega} u\Big(r(1-\theta_j^\omega)k_j + A(\theta_j^\omega k_j + x_j^\omega) + \ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j\Big) \tag{35}$$

subject to

$$\theta_j^\omega k_j \leq -q^\omega x_j^\omega + \ell_j, \tag{36}$$

$$-x_j^\omega \leq \eta k_j, \tag{37}$$

A distressed agent's date 2 consumption in (35) is given by the sum of the return on the long-term assets that were not shored up $r(1-\theta_j^\omega)k_j$, the return on the long-term assets that were shored up but not sold $A(\theta_j^\omega k_j + x_j^\omega)$, and the return on the funds invested at date 1 in the storage technology $\ell_j - q^\omega x_j^\omega - \theta_j^\omega k_j$. (36) is the date 1 budget constraint stating that reinvestment $\theta_j^\omega k_j$ needs to be covered by the sum of ex-ante liquidity hoarding ℓ_j and proceeds of ex-post sale of assets $-q^\omega x_j^\omega$.

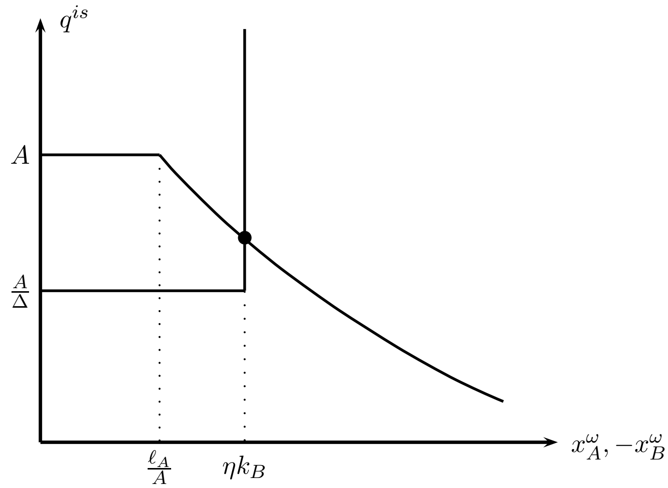

Figure 5: Asset market equilibrium in state (i, s).

(37) says that distressed agents cannot resell more than a fraction η of their initial long-term assets holdings k_j.

The form of the objective in (35) again yields a simple form for a distressed agent's asset supply schedule. For $q^\omega < A/\Delta$, an agent does not want to sell any assets, since the revenue from a sale is lower than the cost of shoring up the asset. At $q^\omega = A/\Delta$, an agent is indifferent between selling any amount it can. Finally, for $q^\omega > A/\Delta$, an agent wants to sell as much as possible. The distressed agents' asset supply curve is therefore horizontal at $q^\omega = A/\Delta$ and vertical at ηk_j for $q^\omega > A/\Delta$.

Asset market clearing requires $x_A^\omega + x_B^\omega = 0$. The equilibrium asset price necessarily satisfies $A/\Delta \leq q^\omega \leq A$. The equilibrium is simply given by $q^\omega = A/\Delta$ and $x_A^\omega = x_B^\omega = 0$ in state (i, i), and by $q^\omega = A$ and $x_A^\omega = x_B^\omega = 0$ in state (s, s). When country j is intact and country $-j$ is distressed (i.e. in states (i, s) and (s, i)), under the condition that $A/\Delta < \frac{\ell_j}{\eta k_{-j}} < A$, the equilibrium takes the form displayed in Figure 5, and equating the asset demand ℓ_j/q^ω with the asset supply ηk_{-j} yields an equilibrium asset price of

$$q^\omega = \frac{\ell_j}{\eta k_{-j}}. \tag{38}$$

The equilibrium features *cash-in-the-market pricing*, as in Allen and Gale (1998), in that the asset price depends positively on the amount of liquidity in the hands of the intact country's buyers. A marginal increase in the amount of liquidity set aside ex-ante by intact agents would push up the asset price by increasing the demand for the asset (shifting the downward sloping part of the demand curve to the right). The equilibrium also exhibits *fire-sales* in that the asset price depends negatively on the amount of illiquid assets thrown on the market by the distressed country's sellers. A marginal decrease in the amount of ex-ante illiquid investment by distressed agents would push up the asset price by reducing the supply of the asset (shifting the vertical portion of the supply curve to the left). Naturally, such movements in the asset price have ex-ante welfare implications similar to movements in the interest rate in the model of section 2.

In equilibrium, the date 1 value function of an intact agent is given by

$$V_i^{\omega}(k_j, \ell_j) = u\Big(Ak_j + \frac{A}{q^{\omega}}\ell_j\Big), \tag{39}$$

and that of a distressed agent is given by

$$V_s^{\omega}(k_j, \ell_j) = u\Big(r(1-\eta)k_j + \Delta(q^{\omega}-1)\eta k_j + \Delta\ell_j\Big). \tag{40}$$

(39) indicates that for an intact agent, illiquid assets yield a return of A, while liquid assets, by allowing the purchase at price q^{ω} of assets that have an ultimate return of A, yield a return of A/q^{ω}. The terms in (40) can be interpreted similarly. For a distressed agent, each unit of liquid assets allows shoring up one unit of illiquid assets, yielding a return Δ, whereas each unit of illiquid assets yields a return of r for the share $1-\eta$ that cannot be sold off, and a return of $\Delta(q^{\omega}-1)$ for the share η that can be sold off (an asset sale brings an extra $q^{\omega}-1$ of date 1 liquidity, whose return is Δ).

A remarkable aspect of the asset market equilibrium is its complete isomorphism with the credit market equilibrium of the model of section 2. For $\eta = \frac{\kappa r}{A}$, the equilibrium of the asset market model corresponds exactly to the equilibrium of the credit market model, when prices are redefined according to $R^{\omega} = \frac{A}{q^{\omega}}$. When $\underline{\eta} < \eta < \bar{\eta}$, for $\underline{\eta} = \frac{\underline{\kappa} r}{A}$ and $\bar{\eta} = \frac{\bar{\kappa} r}{A}$, date 0 decisions in a symmetric competitive equilibrium result in the date 1 asset market equilibrium taking the form displayed in Figure 5. The results derived in sections 3.2 to 4 for the credit market model therefore automatically also apply to the asset market model of this section. This illustrates that the coordination problem between national regulatory authorities that is at the core of this paper does not rely on a particular specification of the market allowing distressed entities to raise funds during a crisis. In the asset market version of the model, global regulation calls for more investment in liquid assets and less investment in illiquid assets ex-ante, with the aim of supporting asset prices during financial crises. Marginally higher asset prices in a crisis redistribute wealth from intact buyers to distressed sellers, and thereby achieve a reduction in the cross-sectional wedges between marginal rates of substitution between goods in states (i, s) and (s, i). National planners do not aim at a reduction of these wedges, but rather try to shift surplus in favor of domestic residents, by pushing down the asset price in states where their residents are buying the asset and pushing it up in states where the residents are selling the asset.

6 Numerical illustration

This section presents a numerical illustration that provides information about the relative magnitude of the different distortions arising in the model.

The parameters are chosen so as to illustrate the key results, subject to satisfying assumptions 1, 2 and 3. Utility is assumed logarithmic. The illiquid project is assumed to deliver a return of $A = 1.5$ when intact and of $r = 0.3$ when distressed. This yields an internal rate of return of

$\Delta = 1.2$ in terms of date 2 goods for a distressed agent at date 1. The fraction of certain output that can be collateralized for loans at date 1 is set to $\kappa = 0.4$. Finally, liquidity shocks are assumed to be uncorrelated across the two countries, and the unconditional probability of a country being distressed is set to $\alpha = 0.13$. The probabilities of the respective states of nature are thus given by $\pi^{ii} = (1-\alpha)^2 \approx 0.757$, $\pi^{is} = \pi^{si} = \alpha(1-\alpha) \approx 0.113$ and $\pi^{ss} = \alpha^2 \approx 0.017$. A time period is interpreted as 2.5 years. The expected net return on illiquid projects is thus 34.4% over 5 years (from date 0 to date 2), i.e. about 6% per year. The unconditional probability of a project getting distressed implies that a crisis occurs on average every 19 years in a given country.

In a competitive equilibrium, investors choose to allocate 9.8% of their portfolio to liquid assets. To achieve constrained efficiency, a global planner would instead require investors to hoard liquidity in the amount of 10.7%. In a nationally regulated equilibrium, countries would only invest 9.1% of their portfolios in liquid assets. Welfare is computed as a permanent (i.e. constant across states of nature) percentage deviation from consumption in the competitive equilibrium. Global regulation results in a welfare gain of 0.24% of permanent consumption relative to the laissez-faire case, while national regulations result in a welfare loss of 0.22% of permanent consumption relative to the laissez-faire case. These welfare gains and losses are seem rather small, but these numbers imply that the losses from failing to coordinate are of the same order of magnitude as the potential benefits of regulation.

It is also instructive to see how prices and allocations during financial crises, i.e. in states (i, s) and (s, i), vary across the same three allocation systems. In the credit market version of the model, the gross interest rate during a crisis rises to 1.1 in the competitive equilibrium, versus 1.0 (the lower bound) in the globally regulated equilibrium and 1.2 (the upper bound Δ) in the nationally regulated equilibrium.

Similarly, in the asset market version, the price of the illiquid asset drops to 1.36 in the competitive equilibrium, versus 1.5 (the upper bound A) and 1.25 (the lower bound A/Δ) in the globally and nationally regulated equilibria, respectively. These numbers illustrate that relatively small changes in the liquidity of ex-ante investment portfolios can have significant impacts on price movements during crises. The liquidation rate (fraction of illiquid projects not shored up in a crisis) amounts to 78.2% in the competitive equilibrium, versus 76.0% and 80.0% in the globally and nationally regulated equilibria, respectively. The consumption of distressed agents drops to 0.359 in a competitive equilibrium, against 0.375 and 0.345 in the globally and nationally regulated equilibria, respectively.

7 Conclusion

This paper studies the international coordination problem inherent to the financial stability objective of banking regulation. It presents a model of systemic risk-taking where the stabilization benefits of macro-prudential regulation are global, but the costs of regulation stemming from distortions in production are incurred domestically. Absent coordination, this public goods problem naturally results in an underprovision of regulation.

Our findings have implications that reach far beyond the framework of the particular model analyzed. They illustrate the intuitive point that in situations where pecuniary externalities may operate across borders, there is a strong case for cooperation in policies whose underlying motivation is to correct such externalities. Hence, they notably imply that the case for international coordination extends to a large part of the growing research agenda that motivates financial crisis prevention policies from a second best perspective in incomplete markets environments (see reviews in Wagner 2009 and Korinek 2011a). The quantitative relevance of the implied coordination problem is likely to depend on the particular frictions and policies under scrutiny, and its assessment is an important area for future research.

At a more abstract level, the paper also shows that, contrary to common beliefs, market power can increase the magnitude of the distortions induced by systemic externalities. Whether market power attenuates or amplifies these distortions crucially depends on the direction of the relevant pecuniary externalities, as well as on whether they are imposed on ex-post identical or ex-post different agents. The non-trivial interplay between distortions arising from market incompleteness and those due to non-competitive market structures is another fruitful avenue for future research.

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A Proofs

Proof of lemma 1

Without loss of generality, consider state (i, s). For part 1., differentiating (9) for $j = A$ and (15) with respect to ℓ_A yields

$$\frac{\partial V_i^{is}(k_A, \ell_A)}{\partial \ell_A} = R^{is} u'\Big(Ak_A + R^{is}\ell_A\Big), \tag{A.1}$$

and

$$\frac{\partial \tilde{V}_i^{is}(k_A, k_B, \ell_A, \ell_B)}{\partial \ell_A} = \Delta u'\Big((1-\kappa)rk_B + \Delta(\ell_A + \ell_B)\Big). \tag{A.2}$$

Now, observe that $\ell_A + \ell_B < k_A$ and $r < 1 < R^{is}$ imply that the following inequality holds

$$\Delta(\ell_A + \ell_B) + r(\ell_A - \ell_B) < \Delta k_A + 2R^{is}\ell_A.$$

Adding and subtracting r on the left-hand side, one obtains

$$\Delta(\ell_A + \ell_B) + rk_B - rk_A < \Delta k_A + 2R^{is}\ell_A.$$

Since $\Delta \equiv A - r$, this implies

$$\Delta\ell_B + (\Delta - R^{is})\ell_A + rk_B < Ak_A + R^{is}\ell_A. \tag{A.3}$$

Together with $R^{is} = \kappa rk_B/\ell_A < \Delta$ and the concavity of $u(\cdot)$, this implies that for given $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.2) is larger than the one in (A.1).

For part 2., differentiating (10) for $j = B$ and (16) with respect to k_B yields

$$\frac{\partial V_i^{is}(k_B, \ell_B, R^{is})}{\partial k_B} = [r + (\Delta - R^{is})\frac{\kappa r}{R^{is}}]u'\Big(rk_B + (\Delta - R^{is})\frac{\kappa rk_B}{R^{is}} + \Delta\ell_B\Big), \tag{A.4}$$

and

$$\frac{\partial \tilde{V}_i^{is}(k_A, k_B, \ell_A, \ell_B)}{\partial k_B} = (1-\kappa)ru'\Big((1-\kappa)rk_B + \Delta(\ell_A + \ell_B)\Big) + \kappa ru'\Big(Ak_A + \kappa rk_B\Big). \tag{A.5}$$

(A.3), $R^{is} = \kappa rk_B/\ell_A < \Delta$ and the concavity of $u(\cdot)$ imply that for given $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.5) is smaller than the one in (A.4).

Proof of lemma 2

For part 1., differentiating (20) with respect to ℓ_A yields

$$\frac{\partial \hat{V}_i^{id}(k_A, k_B, \ell_A, \ell_B)}{\partial \ell_A} = 0. \tag{A.6}$$

Clearly, for any $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.6) is smaller than the one in (A.2).

For part 2., differentiating (21) with respect to k_B yields

$$\frac{\partial \tilde{V}_i^{di}(k_A, k_B, \ell_A, \ell_B)}{\partial k_B} = (1-\kappa) r u'\Big((1-\kappa) r k_B + \Delta(\ell_A + \ell_B)\Big). \tag{A.7}$$

For any $(k_A, k_B, \ell_A, \ell_B)$ in region II, the expression in (A.7) is smaller than the one in (A.5).

Proof of lemma 3

For part 1., we simply observe that the expression in (A.6) is smaller than the one in (A.1). Similarly, for part 2., it is clear that the expression in (A.7) is smaller than the one in (A.4).

Proof of proposition 1

We start by defining the two functions characterizing a symmetric competitive equilibrium and a symmetric globally regulated equilibrium, respectively,

$$
\begin{aligned}
g_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big)+\pi^{is}\Big(A-\frac{\kappa rk}{1-k}\Big)u'\Big((A+\kappa r)k\Big) \qquad \text{(A.8)}\\
& +\pi^{si}\Big[r+\Big(\Delta-\frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}}-\Delta\Big]u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)+\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big),
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{g}_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big)+\pi^{is}\Big[Au'\Big((A+\kappa r)k\Big)-\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \qquad \text{(A.9)}\\
& +\pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk+2\Delta(1-k)\Big)+\kappa ru'\Big((A+\kappa r)k\Big)-\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big]\\
& +\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big).
\end{aligned}
$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE} \in \Big(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big)$, and is implicitly given by $g_{II}(k^{CE}) = 0$. A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k}) = 0$ if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k}) \leq 0$ if $\tilde{k} = \frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k}) \geq 0$ if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\tilde{g}_{II}(k) < g_{II}(k)$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, then either $\tilde{k} \in \Big[\frac{1}{1+\kappa r}, k^{CE}\Big)$ with $\tilde{g}_{II}(\tilde{k}) = 0$, or $\tilde{k} = \frac{1}{1+\kappa r} < k^{CE}$ with $\tilde{g}_{II}(\tilde{k}) < 0$. It follows that showing that $g'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\tilde{g}_{II}(k) < g_{II}(k)$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$ is sufficient to prove that $\tilde{k} < k^{CE}$.

The derivative of $g_{II}(\cdot)$ is given by:

$$
\begin{aligned}
g'_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)^2u''\Big((A-1)k+1\Big)\\
& +\pi^{is}\Big[-\frac{\kappa r(1-k)+\kappa rk}{(1-k)^2}u'\Big((A+\kappa r)k\Big)+\Big(A-\frac{\kappa rk}{1-k}\Big)(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big]\\
& +\pi^{si}\Big\{\Big[r+\Big(\Delta-\frac{\kappa rk}{1-k}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-k}}-\Delta\Big]\Big((1-\kappa)r-2\Delta\Big)u''\Big((1-\kappa)rk+2\Delta(1-k)\Big)\\
& -(\Delta+\kappa r)u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big\}\\
& +\pi^{ss}(r-\Delta)^2u''\Big(rk+\Delta(1-k)\Big).
\end{aligned}
$$

Each single term of $g'_{II}(k)$ is strictly negative for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, so $g'_{II}(k) < 0$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$.

Similarly, the derivative of $\tilde{g}_{II}(\cdot)$ is given by:

$$\begin{aligned}
\tilde{g}'_{II}(k) \quad &\equiv \quad \pi^{ii}(A-1)^2u''\Big((A-1)k+1\Big) \\
&+\pi^{is}\Big[A(A+\kappa r)u''\Big((A+\kappa r)k\Big) - \Delta[(1-\kappa)r-2\Delta]u''\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \\
&+\pi^{si}\Big[[(1-\kappa)r-\Delta][(1-\kappa)r-2\Delta]u''\Big((1-\kappa)rk+2\Delta(1-k)\Big) + \kappa r(A+\kappa r)u''\Big((A+\kappa r)k\Big)\Big] \\
&+\pi^{ss}(r-\Delta)^2u''\Big(rk+\Delta(1-k)\Big).
\end{aligned}$$

Every single term is strictly negative for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, sot $\tilde{g}'_{II}(k) < 0$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$.

It remains to show that $\tilde{g}_{II}(k) < g_{II}(k)$ over $\Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$. Defining $\tilde{\Phi}(k) \equiv \tilde{g}_{II}(k) - g_{II}(k)$, we have

$$\begin{aligned}
\tilde{\Phi}(k) \quad &= \quad -\pi^{is}\Big[\Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big) - \frac{\kappa rk}{1-k}u'\Big((A+\kappa r)k\Big)\Big] \\
&-\pi^{is}\kappa r\Big[u'\Big((1-\kappa)rk+2\Delta(1-k)\Big) - u'\Big((A+\kappa r)k\Big)\Big].
\end{aligned}$$

Lemma 1 established that the terms in brackets are positive for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$, implying that $\tilde{\Phi}(k) < 0$ for $k \in \Big[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big]$. It follows that $\tilde{k} < k^{CE}$, and therefore that $\tilde{\ell} > \ell^{CE}$.

Proof of proposition 2

We start by defining the function characterizing a symmetric nationally regulated equilibrium

$$\begin{aligned}
\hat{g}_{II}(k) \quad &\equiv \quad \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{is}Au'\Big((A+\kappa r)k\Big) \\
&+\pi^{si}\Big[(1-\kappa)ru'\Big((1-\kappa)rk+2\Delta(1-k)\Big) - \Delta u'\Big((1-\kappa)rk+2\Delta(1-k)\Big)\Big] \\
&+\pi^{ss}(r-\Delta)u'\Big(rk+\Delta(1-k)\Big).
\end{aligned} \tag{A.10}$$

Under assumption 3, the value of k in a symmetric competitive equilibrium, k^{CE}, falls in the interior of region II, $k^{CE} \in \Big(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\Big)$, and is implicitly given by $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in A.8). A necessary condition for a globally regulated equilibrium is that $\tilde{g}_{II}(\tilde{k}) = 0$ (with $\tilde{g}_{II}(\cdot)$ defined in A.9) if $\frac{1}{1+\kappa r} < \tilde{k} < \frac{\Delta}{\Delta+\kappa r}$, $\tilde{g}_{II}(\tilde{k}) \leq 0$ if $\tilde{k} = \frac{1}{1+\kappa r}$ and $\tilde{g}_{II}(\tilde{k}) \geq 0$ if $\tilde{k} = \frac{\Delta}{\Delta+\kappa r}$. Similarly, a necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ if $\frac{1}{1+r\kappa} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\Big[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+\kappa r}\Big]$, then we can be a priori in either of the following three situations

- $\hat{k} = \tilde{k} = \frac{1}{1+\kappa r}$, with $\hat{g}_{II}(\hat{k}) \leq 0$, $\tilde{g}_{II}(\tilde{k}) < 0$,
- $\tilde{k} = \frac{1}{1+\kappa r} < \hat{k}$, with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) \leq 0$,
- $\frac{1}{1+\kappa r} < \tilde{k} < k^{CE}$ and $\tilde{k} < \hat{k}$ with $\hat{g}_{II}(\hat{k}) \geq 0$, $\tilde{g}_{II}(\tilde{k}) = 0$.

It follows that showing that $\hat{g}'_{II}(k) < 0$, $\tilde{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > \tilde{g}_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} \geq \tilde{k}$. Moreover, if in addition $\hat{k} > \frac{1}{1+\kappa r}$, then $\hat{k} > \tilde{k}$.

The fact that $\tilde{g}'_{II}(k)$ for $k \in \left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has been established as part of proposition 1. The derivative of $\hat{g}_{II}(\cdot)$ is given by:

$$\begin{aligned}\hat{g}'_{II}(k) \quad \equiv \quad & \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) + \pi^{is} A(A+\kappa r)u''\Big((A+\kappa r)k\Big) \\ & +\pi^{si}[(1-\kappa)r - \Delta][(1-\kappa)r - 2\Delta]u''\Big((1-\kappa)rk + 2\Delta(1-k)\Big) \\ & +\pi^{ss}(r-\Delta)^2 u''\Big(rk + \Delta(1-k)\Big).\end{aligned}$$

Every single term is strictly negative for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$.

Now, defining $\check{\Phi}(k) \equiv \hat{g}_{II}(k) - \tilde{g}_{II}(k)$, we have

$$\check{\Phi}(k) = \pi^{is}\Big[\Delta u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) - \kappa r u'\Big((A+\kappa r)k\Big)\Big] > 0$$

The term in bracket is positive for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, so $\check{\Phi}(k) > 0$ for $k \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. It follows that

- $\hat{k} \geq \tilde{k}$ and $\hat{\ell} \leq \tilde{\ell}$,
- if $\hat{k} > \frac{1}{1+\kappa r}$ then $\hat{k} > \tilde{k}$ and $\hat{\ell} < \tilde{\ell}$.

Proof of proposition 3

Under assumption 3, $k^{CE} \in \left(\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right)$ and $g_{II}(k^{CE}) = 0$ (with $g_{II}(\cdot)$ defined in (A.8)). A necessary condition for a nationally regulated equilibrium is that $\hat{g}_{II}(\hat{k}) = 0$ (with $\hat{g}_{II}(\cdot)$ defined in (A.10)) if $\frac{1}{1+\kappa r} < \hat{k} < \frac{\Delta}{\Delta+\kappa r}$, $\hat{g}_{II}(\hat{k}) \leq 0$ if $\hat{k} = \frac{1}{1+\kappa r}$ and $\hat{g}_{II}(\hat{k}) \geq 0$ if $\hat{k} = \frac{\Delta}{\Delta+\kappa r}$. Note that if $g'_{II}(k) < 0$, $\hat{g}'_{II}(k) < 0$ and $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$, then either $\hat{k} \in \left(k^{CE}, \frac{\Delta}{\Delta+\kappa r}\right]$ with $\hat{g}_{II}(\hat{k}) = 0$, or $\hat{k} = \frac{\Delta}{\Delta+\kappa r} > k^{CE}$ with $\hat{g}_{II}(\hat{k}) \geq 0$. The fact that $g'_{II}(k) < 0$ and $\hat{g}'_{II}(k) < 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ has already been established in the proofs of propositions 1 and 2. It follows that showing that $\hat{g}_{II}(k) > g_{II}(k)$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ is sufficient to prove that $\hat{k} > k^{CE}$.

Defining $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, we have

$$\begin{aligned}\hat{\Phi}(k) \quad &= \quad \frac{\kappa r k}{1-k} u'\Big((A+\kappa r)k\Big) - \Delta \frac{\kappa r(1-k)}{\kappa r k} u'\Big((1-\kappa)rk + 2\Delta(1-k)\Big) \\ &= \quad \frac{\kappa r}{(A+\kappa r)(1-k)} - \frac{\Delta(1-k)}{(1-\kappa)rk^2 + 2\Delta(1-k)k}\end{aligned} \tag{A.11}$$

We now show that under the logarithmic utility assumption, $\hat{\Phi}(k) > 0$ over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$.

We observe that $\hat{\Phi}(k)$ is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$ if and only if the transformed function

$$\hat{\Phi}_T(k) \equiv [(A+\kappa r)(1-k)][(1-\kappa)rk^2 + 2\Delta(1-k)k]\hat{\Phi}(k)$$

is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. Conveniently, $\hat{\Phi}_T(k)$ is quadratic:

$$\hat{\Phi}_T(k) = (1-\kappa)\kappa r^2 k^2 + 2\Delta\kappa r(1-k)k - \Delta(A+\kappa r)(1-k)^2,$$

with a derivative given by

$$\hat{\Phi}'_T(k) = 2(1-\kappa)\kappa r^2 k + 2\Delta\kappa r(1-k) - 2\Delta\kappa r k + 2\Delta(A+\kappa r)(1-k).$$

When evaluated at the bounds $\frac{1}{1+\kappa r}$ and $\frac{\Delta}{\Delta+\kappa r}$, both $\hat{\Phi}_T(k)$ and $\hat{\Phi}'_T(k)$ are strictly positive under assumption 1 that $A \leq \frac{3}{2}$:

$$\begin{aligned}
\hat{\Phi}_T\left(\frac{1}{1+r\kappa}\right) &= \frac{\kappa r^2}{(1+\kappa r)^2}\Big[(1-\kappa) + \Delta\kappa[2-(A+\kappa r)\Big] > 0 \\
\hat{\Phi}_T\left(\frac{\Delta}{\Delta+r\kappa}\right) &= \frac{\Delta\kappa r^2}{(\Delta+\kappa r)^2}\Big[\Delta(1-\kappa) + \Delta\kappa[2\Delta-(A+\kappa r)]\Big] > 0 \\
\hat{\Phi}'_T\left(\frac{1}{1+r\kappa}\right) &= \frac{2\kappa r}{1+\kappa r}\Big[(1-\kappa)r + \Delta[2\kappa r + (A-1)]\Big] > 0 \\
\hat{\Phi}'_T\left(\frac{\Delta}{\Delta+r\kappa}\right) &= \frac{2\Delta\kappa r}{\Delta+\kappa r}\Big[(1-\kappa)r + 2\kappa r + (A-\Delta)\Big] > 0
\end{aligned}$$

Due to linearity, $\hat{\Phi}'_T(\cdot)$ is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. This, together with the strict positiveness of $\hat{\Phi}_T(\frac{1}{1+r\kappa})$ and $\hat{\Phi}_T(\frac{\Delta}{\Delta+r\kappa})$, guarantees that $\hat{\Phi}_T(\cdot)$ is strictly positive over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. In turn, this implies that $\hat{\Phi}(\cdot)$ is strictly positive as well over $\left[\frac{1}{1+r\kappa}, \frac{\Delta}{\Delta+r\kappa}\right]$. It follows that $\hat{k} > k^{CE}$, and therefore $\hat{\ell} < \ell^{CE}$.

Proof of proposition 4

For regulations in country B sufficiently close to a competitive equilibrium (i.e. K_B in the neighborhood of k^{CE}), the date 0 investment decision of private agents in country A satisfies $f(k_A^{CE}, K_B) = 0$, where

$$\begin{aligned}
f(k, K_B) \equiv\; & \pi^{ii}(A-1)u'\Big((A-1)k+1\Big) + \pi^{is}\Big(A - \frac{\kappa r K_B}{1-k}\Big)u'\Big(Ak + \kappa r K_B\Big) \\
& + \pi^{si}\Big[r + \Big(\Delta - \frac{\kappa r k}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa r k}{1-K_B}} - \Delta\Big]u'\Big((1-\kappa)rk + \Delta(2-k-K_B)\Big) \\
& + \pi^{ss}(r-\Delta)u'\Big(rk + \Delta(1-k)\Big).
\end{aligned} \tag{A.12}$$

The partial derivatives of $f(k, K_B)$ are given by

$$\begin{aligned}\frac{\partial f}{\partial k} &= \pi^{ii}(A-1)^2 u''\Big((A-1)k+1\Big) \\ &+\pi^{is}\Big[-\frac{\kappa r K_B}{(1-k)^2}u'\Big(Ak+\kappa r K_B\Big)+\Big(A-\frac{\kappa r K_B}{1-k}\Big)Au''\Big(Ak+\kappa r K_B\Big)\Big] \\ &\pi^{si}\Big\{-\frac{\Delta(1-K_B)}{k^2}u'\Big((1-\kappa)rk+\Delta(2-k-K_B)\Big) \\ &+\Big[r+\Big(\Delta-\frac{\kappa rk}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-K_B}}-\Delta\Big][(1-\kappa)r-\Delta]u''\Big((1-\kappa)rk+\Delta(2-k-K_B)\Big)\Big\} \\ &+\pi^{ss}(r-\Delta)^2u''\Big(kr+\Delta(1-k)\Big),\end{aligned}$$

and

$$\begin{aligned}\frac{\partial f}{\partial K_B} &= \pi^{is}\Big[-\frac{\kappa r}{1-k}u'\Big(Ak+\kappa r K_B\Big)+\Big(A-\frac{\kappa r K_B}{1-k}\Big)\kappa r u''\Big(Ak+\kappa r K_B\Big)\Big] \\ &\pi^{si}\Big\{-\frac{\Delta}{k}u'\Big((1-\kappa)rk+\Delta(2-k-K_B)\Big) \\ &+\Big[r+\Big(\Delta-\frac{\kappa rk}{1-K_B}\Big)\frac{\kappa r}{\frac{\kappa rk}{1-K_B}}-\Delta\Big](-\Delta)u''\Big((1-\kappa)rk+\Delta(2-k-K_B)\Big)\Big\}.\end{aligned}$$

When evaluated at $(k, K_B) = (k^{CE}, k^{CE})$, each of the individual terms making up $\frac{\partial f}{\partial k}$ and $\frac{\partial f}{\partial K_B}$ are negative, and therefore $\frac{\partial f}{\partial k}$ and $\frac{\partial f}{\partial K_B}$ are negative. By the implicit function theorem

$$\frac{dk_A^{CE}}{dK_B} = -\frac{\frac{\partial f}{\partial K_B}}{\frac{\partial f}{\partial k}} < 0.$$

Hence k_A^{CE} is decreasing in K_B.

Proof of proposition 5

In the neighborhood of the symmetric competitive equilibrium, country A's national planner payoff can be written as

$$\begin{aligned}\Pi(k_A, k_B) &\equiv \sum_{\omega\in\Omega}\pi^{\omega}\hat{V}^{\omega}(k_A, k_B, 1-k_A, 1-k_B) \qquad \text{(A.13)} \\ &= \pi^{ii}u\Big(Ak_A+(1-k_A)\Big)+\pi^{is}u\Big(Ak_A+\kappa r k_B\Big)+\pi^{si}u\Big((1-\kappa)rk_A+\Delta(1-k_A+1-k_B)\Big) \\ &+\pi^{ss}u\Big(rk_A+\Delta(1-k_A)\Big)\end{aligned}$$

The cross derivative is given by

$$\frac{\partial^2\Pi}{\partial k_A\partial k_B} = \pi^{is}\kappa r A u''\Big(Ak_A+\kappa r k_B\Big)+\pi^{si}(-\Delta)[(1-\kappa)r-\Delta]u''\Big((1-\kappa)rk_A+\Delta(1-k_A+1-k_B)\Big)$$

This cross derivative is unambiguously negative. It follows that the regulators' actions are strategic substitutes.

Proof of proposition 6

Note that $\frac{\partial W}{\partial K_B}$ can be written as

$$\begin{aligned}\frac{\partial W}{\partial K_B} &= \pi^{is}\Big[\frac{\kappa r k^{CE}}{1-k^{CE}}u'\Big((A+\kappa r)k^{CE}\Big) - \Delta\frac{\kappa r(1-k^{CE})}{\kappa r k^{CE}}u'\Big([(1-\kappa)r-2\Delta]k^{CE}+2\Delta\Big)\Big] \\ &= \pi^{is}\hat{\Phi}(k^{CE})\end{aligned}$$

where $\hat{\Phi}(k) \equiv \hat{g}_{II}(k) - g_{II}(k)$, for $g_{II}(\cdot)$ and $\hat{g}_{II}(\cdot)$ defined in (A.8) and (A.10), respectively. The proof of proposition 3 established that under the logarithmic utility assumption, $\hat{\Phi}(k)$ is strictly positive over $\left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$. Since $k^{CE} \in \left[\frac{1}{1+\kappa r}, \frac{\Delta}{\Delta+\kappa r}\right]$, it follows that $\hat{\Phi}(k^{CE}) > 0$, and therefore that $\frac{\partial W}{\partial K_B} > 0$. Proposition 4 established that $\frac{dk_A^{CE}}{dK_B} < 0$, and it was argued in the text that $\frac{\partial W}{\partial k_A} < 0$. Hence the direct effect and the indirect effect of a change in country B's regulation on country B's welfare work in the same direction:

$$\frac{dW}{dK_B} = \underbrace{\frac{\partial W}{\partial K_B}}_{(+)} + \underbrace{\frac{\partial W}{\partial k_A}\frac{dk_A^{CE}}{dK_B}}_{(+)} > 0.$$

Proof of proposition 7

The proof of proposition 6 showed that under the logarithmic utility assumption, $\frac{\partial W}{\partial K_B} > 0$. Since $\frac{\partial \Pi}{\partial k_A} = \frac{\partial W}{\partial K_B}$, it follows that $\frac{\partial \Pi}{\partial k_A} > 0$. Proposition 4 established that $\frac{dk_A^{CE}}{dK_B} < 0$. Further, it was argued in the text that $\frac{\partial W}{\partial k_A} < 0$. Since $\frac{\partial \Pi}{\partial K_B} = \frac{\partial W}{\partial k_A}$, it follows that $\frac{\partial \Pi}{\partial K_B} < 0$. Hence the direct effect and the indirect effect of a change in country B's regulation on country A's welfare work in the same direction:

$$\frac{d\Pi}{dK_B} = \underbrace{\frac{\partial \Pi}{\partial k_A}\frac{dk_A^{CE}}{dK_B}}_{(-)} + \underbrace{\frac{\partial \Pi}{\partial K_B}}_{(-)} < 0.$$